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Mathematics Vocabulary and English Learners: A Study of Students' Mathematical Thinking

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ABSTRACT

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This study examined the mathematical thinking of English learners as they were taught mathematics vocabulary through research-based methods. Four English learners served as focus students. After administering a pre-performance assessment, I taught a 10-lesson unit on fractions. I taught mathematics vocabulary through the use of a mathematics word wall, think-pair-shares, graphic organizers, journal entries, and picture dictionaries. The four focus students were audio recorded to capture their spoken discourse. Student work was collected to capture written discourse.

Over the course of the unit, the four focus students used the mathematics vocabulary words that were taught explicitly. The focus students gained both procedural and conceptual knowledge of fractions during this unit. Students also expressed elevated confidence in their mathematics abilities.

Keywords: mathematics, vocabulary, mathematics vocabulary, English learners, inquiry teaching

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Introduction

Statement of Problem

Language is our major means of communication, and as teachers we teach through the medium of language (Thompson & Rubenstein, 2000). In order to communicate effectively, one must know words, or vocabulary (National Early Literacy Panel, 2008). In their 1984 study, Nagy and Anderson determined that, at that time, approximately 88,500 distinct words existed in printed school English. They estimated that students encounter between 3,000 and 4,000 completely new vocabulary words each school year. Understanding the meanings of words is central to verbal thought. Therefore, the more words a person deeply understands, the more freedom of thought he or she has.

Learning and retaining the meanings of many new vocabulary words can be challenging. This challenge is intensified when the new vocabulary is part of a complex content area such as mathematics. The use of mathematics vocabulary is concentrated primarily in school settings, while basic interpersonal communicative skills (BICS), or common English, is used in almost every aspect of a person's life (Cummins, 2003; Thompson & Rubenstein, 2000). As difficult as learning academic language, or cognitive academic language proficiency (CALP) is for native speakers, English learners (EL) find it even more challenging (Cummins, 2003). ELs typically acquire BICS within two years of exposure in a second language environment, while they usually take five to seven years to acquire CALP (Cummins, 2003).

The American classroom is changing. According to Helman (2008), the number of students in America who speak a language other than English at home has more than doubled in the last 30 years, and nearly three million children in this nation, or about 4% of the school-age population, speak English with difficulty (Helman, 2008). As our classrooms change, we must also adjust the way we teach to serve the EL population.

Learning the English vocabulary necessary to succeed in school is crucial for ELs. "The ability to read and comprehend English-language material is of major consequence for ELs in American schools and society. Becoming a skilled English reader improves access to education and, by extension, the benefits of the larger society" (Proctor, August, Carlo, & Snow, 2005, p. 254). The needs of EL students in our nation's schools cannot be ignored. Because vocabulary has been found to be essential to comprehension, teachers should use research-based practices to teach vocabulary (Blachowicz, 1985). If research-based instructional practices are used in teaching mathematics vocabulary, students are also likely to benefit from increased mathematical understanding (Coggins, Kravin, Coates, & Carroll, 2007).

Statement of the Purpose

The purpose of this study was to examine the mathematical thinking of ELs as they were taught mathematics vocabulary in a rich context. Research-based instructional strategies for teaching vocabulary were implemented in an elementary mathematics class that used an inquiry approach.

Research Question

I conducted this study while teaching a mathematics unit on fractions. I chose the topic of fractions because it is particularly rich in vocabulary. Also, fraction concepts are often difficult for students to learn. Throughout this fractions unit, I gathered data to answer the research question of this study:

> What happens to the mathematical thinking of English learners when they are taught mathematics vocabulary through research-based instructional strategies?

Limitations

This study had various limitations. One limitation is that this was my first time to implement an entire unit of mathematics inquiry. Previously, I had only taught inquiry lessons sporadically. My lack of familiarity with inquiry may have influenced my ability to focus on the vocabulary and the vocabulary instructional methods. Another limitation is that I received consent/assent forms from only 12 of my 23 students. I originally planned to form collaborative groups of four students each, but out of necessity, formed groups of only three. This formation of groups limited my implementation of strategies such as think-pair-shares. Also, because this study addressed only vocabulary of fractions, similar outcomes may not be seen in other content areas.

Definition of Terms

Several terms or abbreviations that will be used in this paper are defined below. While some of these terms are standard and have widely accepted definitions in the literature (e.g., BICS and CALP [Cummins, 2003]), others have been operationally defined specifically for the purposes of my research (e.g., mathematics vocabulary).

BICS. Basic interpersonal communicative skills, or conversational language (Cummins, 2003).

CALP. Cognitive academic language proficiency, or language used in academic settings (Cummins, 2003).

Conceptual understanding. Flexible "understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson, Siegler, & Alibali, 2001, pp. 346–347). In other words, conceptual understanding in mathematics involves knowledge of the mathematical concepts and the ways ideas connect to each other.

Direct teaching. A teacher-centered approach. It can be scripted and often involves step-by-step instruction (American Institutes for Research, 2004).

Discourse. Oral or written communication.

EL. English learner, learners who speak a language other than English in their homes.

Explicit teaching. A method in which concepts being taught are made transparent to the learner in the lesson. Teachers are deliberate in making sure what they are teaching is clear (Serafini, 2004).

> Explicit instruction includes inquiry based learning where the teacher guides students to conceptual understanding of a particular topic, deliberate use of questioning strategies, and a variety of other tools that are explicitly designed to lead students towards [*sic*] a conceptual understanding of a concept or mastery of a procedural skill (D. Suddreth, personal communication, September 22, 2009).

Inquiry learning. A method in which students take active roles in constructing new knowledge through seeking answers to questions and problems.

L1. A person's first language.

L2. A person's second language.

Mathematics vocabulary. For this study, mathematics vocabulary consists of words with meanings that are exclusively mathematical, words with multiple meanings, and general words. Words with exclusively mathematical meanings are technical words that are used only in mathematical situations, such as *integer* and *polygon*. Mathematics vocabulary words with multiple meanings have at least one mathematical meaning but may also have meanings unrelated to mathematics. These other meanings may be familiar to students because they relate to their everyday lives. For example, a student may know a common meaning for the word *area*, but she or he may not know the mathematical meaning. The last category consists of general words that are common and familiar to many young children and typically have exclusively mathematical meanings, such as *triangle*.

Procedural knowledge. "The ability to execute action sequences to solve problems" (Rittle-Johnson, et al., 2001, p. 346). Students with mathematics procedural knowledge are able to carry out the step-by-step processes involved in solving a mathematics problem. A person can know a procedure to solve a mathematics problem without understanding why that procedure works.

Research-based teaching. Teaching based on the results and recommendations of research that is commonly accepted and valued within a field of study.

Token. The total number of words in a passage, including repeated use of words.

Types. The number of unique words in a passage.

Chapter 2

Review of Literature

The instructional strategies I will use in this study are based on the results and recommendations of research. In order to find strategies suggested by researchers, I have examined articles written about general vocabulary instruction, English learners and general vocabulary instruction, mathematics vocabulary, and English learners and mathematics vocabulary. The following section details my findings and provides an explanation for my use of specific instructional strategies.

Vocabulary Instruction

Beck and McKeown (1985) stated, "One of the cornerstones of literacy is acquiring vocabulary" (p. 12). However, there is some disagreement regarding how vocabulary should be taught in the schools. Most researchers agree with Biemiller and Boote (2006) that teaching vocabulary in rich context is more effective than teaching vocabulary without context. One debate seems to be between whether teachers should teach word meanings directly or whether words even need to be taught since vocabulary is acquired indirectly through wide reading and listening. The National Early Literacy Panel (2008) claimed that most words are learned indirectly; however, some vocabulary should be taught directly. Beck and McKeown (1985) developed a system to help teachers decide what sorts of vocabulary should be taught directly. (See the section entitled *Mathematics Instruction* for more detail.)

Laufer (2003) similarly described two routes to acquiring new vocabulary using different terms: incidental and intentional. Incidental vocabulary acquisition refers to learning new vocabulary indirectly as a by-product of another activity (Laufer, 2003). The purpose of these activities is not to learn new vocabulary, but through the activity new vocabulary may be learned anyway. Reading is an activity that facilitates incidental vocabulary acquisition. A person may learn the meaning of new vocabulary words through reading, even though his or her goal may not have been to learn new vocabulary. Intentional vocabulary acquisition, conversely, refers to setting out purposefully to learn the meaning of new words (Laufer, 2003), which can be done through direct teaching. There are far too many words to teach them all directly, but Marzano and Marzano (1988) speculated that about 400 words can be directly taught each school year between grades 3 and 9. (Guidelines for selecting words to be taught in mathematics are discussed in the section titled *Mathematics Vocabulary*.)

Beck and McKeown (1985) stressed some important factors in strong vocabulary instruction: frequent encounters with the new words, richness of instruction, and the extension of the activities beyond the classroom setting. In 2002, Beck, McKeown, and Kucan called for a robust approach to vocabulary instruction: "A robust approach to vocabulary involves directly explaining the meanings of words along with thought-provoking, playful, and interactive follow-up" (p. 2). The National Early Literacy Panel (2008) stated:

> Children learn words best when they are provided with instruction over an extended period of time and when that instruction has them work actively with the words. The more students use new words and the more they use them in different contexts, the more likely they are to learn the words. (p. 31)

For a child to have learned a word, he or she must be able to use it correctly in various circumstances. The following section continues the discussion of vocabulary instruction with a focus on how English learners learn general vocabulary.

Mathematics Vocabulary Instruction

Murray (2004) stated, "Deliberate and careful attention to acquiring and using the vocabulary of mathematics, with its wondrously specific technical language, is a must" (p. 1). Thompson and Rubenstein (2000) agreed, asserting that understanding the language of mathematics is vital for students, and they encouraged teachers to give specific attention to mathematics vocabulary. Ball and Bass (2003) posited, "Mathematical language is central to constructing mathematical knowledge; it provides resources with which claims are developed, made, and justified" (p. 33).

Vocabulary selection. As mentioned previously in this paper, Beck and McKeown (1985) developed a three-tier system to help teachers determine which words should be explicitly taught and which words are likely to be developed incidentally. Tier 1 words are basic words that typically do not need to be addressed in a classroom setting. Many of the words taught explicitly in school will be tier 2 words, or high frequency words for mature language users (Beck & McKeown, 1985). A portion of the words taught in a school year will be tier 3 words, or low frequency, domain-specific words. Some mathematics vocabulary terms fall into tier 3 because these words belong to a specific content area and do not occur often outside the domain of mathematics (Beck, McKeown, & Kucan, 2002). However, while much mathematics vocabulary is technical and unfamiliar, many mathematics vocabulary words are common words that are familiar to children in other contexts but have unfamiliar mathematical meanings. For example, many children know meanings for the words *odd*, *difference*, *product*, and *positive*; however, they may only know a common meaning, not a mathematical meaning (Monroe & Panchyshyn, 1995/96).

Many mathematics vocabulary words are unfamiliar to students because they are rarely seen or heard by students in their everyday lives. Students often have limited background knowledge for these vocabulary words (Monroe & Orme, 2002; Thompson & Rubenstein, 2000). Schell (1981) stated, "Meaningful interpretations of many mathematical words and symbols require considerable content knowledge on the part of the student before proper meaning can be attained" (p. 2). Therefore, before vocabulary should be taught, students need to develop conceptual understanding of the topic being explored (Thompson & Rubenstein, 2000). Schell (1981) explained that mathematical abstractions can become much more meaningful if students have experiences through the use of concrete objects. Consequently, while many authorities encourage teaching critical words before a lesson (Flanigan & Greenwood, 2007; National Early Literacy Panel, 2008), preteaching mathematics vocabulary words before a unit or lesson may not be the best way for students to develop understanding of these words (Schell, 1981; Thompson & Rubenstein, 2000).

There are many ways that students can have concrete mathematical experiences in order to deepen their mathematics vocabulary knowledge. Students should participate in hands-on exploration of concepts to give them concrete experiences (Thompson & Rubenstein, 2000). This exploration, particularly combined with opportunities to discuss their ideas with peers, will contribute to a deeper mathematical understanding of the concepts and a better grasp of the vocabulary that is attached to those concepts than if the students learned the same material through reading the textbook (Coggins et al., 2007). Schell (1981) urged teachers to provide students with opportunities to do more than just memorize the definitions of mathematical words. She further asserted that memorizing vocabulary without having a deep conceptual understanding of what the words mean will not help students in their pursuit of mathematical understanding. In order to communicate effectively using mathematics vocabulary, students need to have a deep understanding of the vocabulary words they are using rather than simply reciting a memorized definition of the words (Carter & Dean, 2006). A child may know the name or label for an idea without sufficiently understanding the underlying concepts.

Murray (2004) encouraged teachers to allow students initially to discuss mathematical concepts using informal, familiar language. These informal discussions contribute to the feeling of ownership of the language. Also, by letting students struggle to find words to explain concepts, they realize the need for precise words to describe situations adequately (Murray, 2004). This struggle to find precise words may lead the students to be more willing to learn and use formal mathematics vocabulary. Coggins et al. (2007) stated, “Mathematics, as an academic discipline, is made up of concepts that are most effectively discussed with proficiency in academic language” (p. 15). Students likely will see this need if they first are allowed to use imprecise language through informal conversations about mathematics.

Instructional strategies. Monroe and Orme (2002) determined that a combination of explicit teaching and rich, meaningful context may be a successful way to teach mathematics vocabulary. Many researchers have suggested instructional strategies to teach mathematics vocabulary. While some researchers may claim that a given vocabulary strategy is effective in all contexts, this section explores some strategies that have been deemed effective for teaching mathematics vocabulary. I will be using the following strategies to teach the unit on fractions to my sixth grade mathematics class. These are well-recognized vocabulary strategies that are generally accepted as being good strategies but are not specific to ELs. I previously discussed guiding principles for teaching vocabulary to ELs; therefore, I chose these specific strategies because they meet those principles.

Murray (2004) recommends using a think-pair-share approach to allow students time to think about and use mathematics vocabulary. When students are faced with a problem or question, they first think about it on their own. They then discuss their thoughts with a partner. When the students feel confident with what they have discussed, they share their ideas with a larger group or the whole class (Murray, 2004). Coggins et al. (2007) also encouraged the use of think-pair-shares, saying they increase the opportunities students have to speak and express their ideas during class. Speaking plays a large role in helping students acquire and feel comfortable using mathematics vocabulary. Along with Murray (2004) and Coggins et al. (2007), Thompson and Rubenstein (2000) stressed the importance of students' owning the language and having them do the talking. By allowing students to have conversations about the mathematics they are learning, teachers give the students an opportunity to use mathematics vocabulary words.

Journal writing is another effective way for students to work through their ideas using their new mathematics vocabulary (Thompson & Rubenstein, 2000). Murray (2004) explained that journals give students an opportunity to practice using new mathematics vocabulary, and journals are often "the original record of any vocabulary that we encounter during problem solving, minilessons with note taking, and daily reflections" (p.16). Journals can be used not only as effective learning tools for students, but can also help inform teachers and guide instruction (National Council of Teachers of Mathematics, 2000).

In addition to think-pair-shares and journals, graphic organizers have been deemed effective tools for teaching mathematics vocabulary (Monroe & Orme, 2002; Thompson & Rubenstein, 2000). "Graphic organizers are visual structures that make it possible to organize words, ideas, information, and so on to further learning goals such as understanding, communicating, and remembering" (Coggins et al., 2007, p. 68). Examples of graphic organizers include t-charts, Venn diagrams, and word maps (Coggins et al., 2007; Monroe & Orme, 2002).

Murray (2004) encouraged teachers to use a mathematics word wall. Teachers should "introduce each word wall entry through a specific, interactive instructional process and refer frequently to individual word wall cards and posters" (Coggins et al. 2007, p. 24). Murray (2004) also proposed that students keep personal word wall charts. Students can create their own picture dictionaries using the words from the word wall to help them retain meanings (Thompson & Rubenstein, 2000). These activities require the students to think frequently about the terms they are learning and using while studying mathematics (Murray, 2004).

English Learners and General Vocabulary

In 2005, Proctor, August, Carlo, and Snow stressed the need for more research on the reading processes of ELs as they learn to read English. "Clearly, more L2 [second language] intervention research should be building on the well-established link between vocabulary knowledge and reading comprehension in both L1 [first language] and L2 populations to explore ways to promote vocabulary knowledge for comprehension" (Procter et al 2005, p. 254). Joshi (2005) concurred that vocabulary and comprehension have a close relationship and that individuals struggle to understand written text if they have poor vocabularies.

Helman (2008) stated, "Knowing words in a language is a key component to understanding text and being able to produce it—reading and writing" (p. 211). August and Shanahan (2006) proposed that oral language proficiency in English and English reading comprehension are positively correlated. Hence, the more English words a person knows and understands, the more easily she or he can comprehend English texts. This relationship could pose some problems for students who know very few English words. However, if students have large vocabularies in their first language, the outlook is very promising. In 1984, Cummins found that students who have well-developed literacy skills in their first language will likely acquire a second language more quickly than students who have not developed literacy skills in their native language. Proctor, August, Carlo, and Snow (2006) conducted a study with ELs who are native Spanish speakers and found that their Spanish vocabulary knowledge enhanced their English reading outcomes.

Clearly, teachers should strive to help all students improve their vocabularies. However, this emphasis is critical for ELs. The question is: How should we do it? Much research has been conducted that supports the act of reading as the main source of vocabulary growth in an individual's first language (Laufer, 2003; Proctor et al., 2005). "The number of words that people acquire in their L1 is too vast to be accounted for by direct teaching of vocabulary" (Laufer, 2003, pp. 567–568). While reading is considered an excellent way for native-English speakers to improve their vocabularies, such incidental learning may not be accessible for EL students. "Though incidental learning is well established as one route to vocabulary acquisition for native speakers, there has been considerable discussion among L2 researchers and practitioners as to whether it occurs as robustly for students learning English as a second or foreign language" (Proctor et al., p. 254).

Learning vocabulary through reading. The process of learning and retaining new vocabulary words from reading text is difficult, especially for individuals for whom the text is written in the L2 (Laufer, 2003; Pulido, 2007). First, students must notice that a word is unknown to them. That step alone can be difficult, considering all of the homonyms and similar lexical forms in the English language (Laufer, 2003; Proctor et al., 2005). If students are successful in recognizing that they have encountered an unfamiliar word, they will then attempt to infer the meaning of the word (Pulido, 2007). Studies show that words with meanings that are easily guessed within the context are usually not retained as well as words that take more effort to figure out (Laufer, 2003). These researchers demonstrate that it is difficult for EL students to be successful in significantly increasing their vocabularies through independent reading alone.

The acquisition of everyday language (BICS) gained through reading text provides a language base in which students can fit their learning of content area vocabulary. Without this language base, students may struggle with non-academic words in addition to the new academic language they are to learn. The more time and energy students can devote to learning this academic language, the more likely they are to understand it. However, if students are focused on trying to understand the everyday vocabulary, they will not be able to devote sufficient time to the academic vocabulary.

Breadth and depth of vocabulary knowledge. If teachers cannot expect their EL students to pick up vocabulary easily through reading, what should teachers do to help these students? The answer to this question partially depends on the teacher's goals for the students. Ouellette (2006) distinguished between breadth and depth of vocabulary knowledge. This researcher found that decoding performance and visual word recognition can be predicted by students' breadth of vocabulary knowledge. In other words, the more words students are at least familiar with, the more words they will be able to decode or recognize with ease. However, depth of vocabulary knowledge is a better predictor for reading comprehension. Therefore, if the goal is to improve reading comprehension, just being familiar with a lot of words is not enough. Students need to have a deep understanding of the meanings of words, including understanding multiple meanings for words, in order to improve their reading comprehension (Ouellette, 2006).

Proctor et al. (2005) stated that positive changes in vocabulary knowledge have direct effects on reading comprehension.

While the quantity of reading required in a mathematics classroom is typically not as high as in other subject areas, the reading that is required is often more difficult. The text is conceptually dense and laden with unfamiliar vocabulary. Schell (1981) stated, "Research indicates that mathematics material is the most difficult content area material to read with more concepts per word, per sentence, and per paragraph than any other area" (p. 2). Schell also suggested that an individual's low reading level in narrative material can contribute to problems with reading mathematics (1981). Therefore, recognizing the relationship between reading nonmathematics texts and mathematics texts is important. "Students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics clearly reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" (National Council of Teachers of Mathematics, 2000, p. 60).

As discussed earlier, reading and other incidental vocabulary acquisition activities may not be the most successful ways for ELs to gain and retain new vocabulary. Studies have shown that an intentional approach to teaching vocabulary is more likely to meet the needs of ELs (Laufer, 2003; Proctor et al., 2005). Paribakht and Wesche (1997) conducted a study in which they compared learners in two reading condition groups. One condition consisted of reading a text and then completing several vocabulary exercises corresponding to that text. The other condition involved reading only. Based on the number of words that were acquired, the group who completed the vocabulary exercises in addition to reading significantly outperformed the group that only read. In 1994, Knight compared groups of learners who read the same text, but one group used dictionaries to look up unfamiliar words. The group who used dictionaries learned more words than the group who did not. Laufer (2003) determined that word-focused tasks are more effective for vocabulary gains than reading only. Therefore, if teachers can design tasks that clearly require students to learn the meanings of words, the students will be more successful in understanding and retaining the new vocabulary words.

Proctor et al. (2005) stated, “Research on potentially effective practices in improving vocabulary knowledge among [the EL] population suggests targeting depth and breadth of vocabulary knowledge, teaching cognate awareness, and supplying engaging texts” (p. 254). Cognates are words that have similar spellings and meanings in multiple languages. Teaching children to be aware of cognates from their native language likely will facilitate their vocabulary growth. However, if students are not familiar with the cognates in their native language, pointing out the similarities between the word in their native language and English likely will not be helpful.

Helman (2008) found that vocabulary lessons that present words in meaningful, interesting texts are often successful. Providing vocabulary-focused tasks alongside the reading of meaningful texts will allow for even more vocabulary knowledge growth (Laufer, 2003). These tasks will contribute to greater depth of vocabulary knowledge than will reading alone, resulting in increased comprehension (Procter et al., 2005).

English Learners and Mathematics Vocabulary

English learners often acquire BICS within one or two years of trying to learn English, while CALP usually takes at least five years to acquire (Cummins, 2003). Therefore, ELs may learn the common meanings of words such as *odd* and *area* early in their journey of learning English, but they may find it difficult to apply the mathematical meanings to such words. Teachers should help children make the distinction between common meanings and academic meanings of these kinds of words. Such instruction is especially helpful for ELs, who may not be aware that a specific word has more than one meaning (Coggins et al., 2007).

Because social language is typically acquired before academic language, Coggins et al. (2007) encouraged the use of conversational English in mathematics classrooms. In 2000, the National Council of Teachers of Mathematics stated, "Children need introductions to the language and conventions of mathematics, at the same time maintaining a connection to their informal knowledge and language. They should hear mathematical language being used in meaningful contexts" (p. 74). Although most ELs acquire BICS within 2 years, they still may lack knowledge of many common words. Coggins et al. (2007) encouraged teachers to arrange collaborative learning groups so that students of varying language proficiency work together. This classroom arrangement allows ELs to become more aware of mathematics language as well as common, everyday language, as native-English speakers use mathematics language both formally and informally (Coggins et al., 2007). Further, this conversational use of mathematics language also supports the learning of mathematical concepts. Coggins and associates supported using a problem solving approach to mathematics with ELs because such an approach encourages communication among students.

While Coggins et al. (2007) encouraged the use of conversational language in mathematics classrooms, they also recognized the importance of teaching mathematics vocabulary. If students have previously learned the mathematical concepts being taught in their native language, teaching cognates may be beneficial (Coggins et al., 2007). Making connections between L1 and L2 when discussing mathematics problems "can enhance the depth of understanding and make it possible for English learners to think about mathematical relationships in the primary language" (Coggins et al., 2007, p. 21).

ELs can learn mathematics vocabulary through various instructional strategies. While reading teachers often encourage teaching vocabulary words before students read a text with those words, this strategy is not encouraged in mathematics (Coggins et al., 2007). As stated earlier, it is important for students to have conceptual understanding before labels are attached (Coggins et al.; Schell, 1981; Thompson & Rubenstein, 2000). Another strategy that many teachers think is helpful, although it is actually detrimental, is emphasizing key words in story problems. For example, these teachers think that if you teach children to subtract every time they see the word *fewer* in a story problem, it will simplify the process, making it easier to solve the problem. This practice may be especially tempting for teachers of ELs. However, this approach often leads to missing the meaning of the word problem and can be harmful to a student's mathematical understanding. Focusing on the mathematical relationships that exist in the problems would be more helpful (Coggins et al., 2007). I taught the fractions unit using specific strategies that have been shown to be successful with EL students.

Chapter 3

Methods

I used an action research approach as I conducted my study. Action research is the process of examining situations in real classrooms and schools in order to develop a plan to improve those situations (Johnson, 2002). This action research design helped me answer my research question:

> What happens to the mathematical thinking of English learners when they are taught mathematics vocabulary through research-based instructional strategies?

In this study, students' discourse and the representations they created served as the lenses for examining their mathematical thinking. I analyzed students' mathematical thinking as they solved problems and used mathematics vocabulary. The study design is described more specifically in the section titled *Design*.

Because I am specifically interested in the mathematical thinking of my students, it is important for me to explain what I mean by *mathematical thinking*. Harel and Sowder (2005) differentiated between mathematical understanding and mathematical thinking by saying:

> In our usage, the phrase *way of understanding*, conveys the reasoning one applies in a local, particular mathematical situation. The phrase *way of thinking*, on the other hand, refers to what governs one's ways of understanding, and thus expresses reasoning that is not specific to one particular situation but to a multitude of situations. A person's ways of thinking involve at least three interrelated categories: beliefs, problem-solving approaches, and proof schemes. (p. 31)

Although I use the phrase *mathematical thinking* in this study, I jointly analyzed both mathematical understanding and mathematical thinking as described by Harel and Sowder

(2005). I examined students' mathematical processes, content knowledge, and other indicators of mathematical understanding that emerged throughout the unit. These indicators yielded patterns that gave insights into students' mathematical thinking.

Participants and Context

The students participating in this study attended a Title I elementary school in Utah. During the 2009–2010 school year, 72% of students in that school received free or reduced-price lunch. Of the 566 students at the school, 244 were Caucasian, 266 were Hispanic, 27 were Pacific Islander, 15 were Native American, seven were Asian, six were African-American, and one was classified in a category called *Other*. The school made adequate yearly progress for the 2008–2009 school year; however, it did not make adequate yearly progress for the 2007–2008 school year (Utah State Office of Education, 2009). The school district in which this school is located adopted the Scott Foresman-Addison Wesley mathematics program in 2007 (Charles, Crown, & Fennell, 2008) for use in grades 3–6.

Eight of the 23 students in this sixth grade mathematics class lived in homes where a language other than English was spoken. At some point all eight had been members of the school English for Students of Other Languages (ESOL) program provided by the school. Two of these students previously had been exited from the program because their scores indicated that they no longer needed ESOL services; one was being monitored for exiting at the time I conducted this study. The remaining five all scored in the intermediate range on the Utah Academic Language Proficiency Assessment (UALPA). The UALPA includes sections for speaking, listening, reading, and writing (Utah State Office of Education, 2007). Scores on the UALPA fall into one of five ranges: pre-emergent, emergent, intermediate, advanced, or fluent

(Utah State Office of Education, 2007). The five students in my class who scored in the intermediate range on the UALPA received ESOL services during the 2009–2010 school year.

Of the five students receiving ESOL services at the time of the study, four returned consent/assent forms. These four students became my focus students. Pseudonyms are used throughout this paper to identify these students. Kyle and Laura are Native American and speak Navajo in their homes. Megan and Yolanda are Hispanic and speak Spanish in their homes. Laura, Megan, and Yolanda lived in Utah the previous year and took the Utah Mathematics Criterion-Referenced Test (CRT) in fifth grade. (This test is not publicly available, but reports of school data are available at Utah State Office of Education, 2010). Students can earn a score of 1, 2, 3, or 4 on the Utah CRTs. A score of 1 indicates minimal understanding of the mathematics concepts. A score of 2 indicates partial understanding of these concepts. A score of 3 is a passing score and indicates sufficient understanding. A score of 4 indicates substantial understanding. Laura scored 1 on the fifth grade mathematics CRT, and Megan and Yolanda both scored 2s; none of the three received passing scores. Because Kyle did not live in Utah while he was in fifth grade, CRT scores were unavailable for him. Prior to the study, however, Kyle had demonstrated consistently high levels of mathematics performance in the classroom for the first several months of the school year.

While only four EL students were selected as my focus students, other students in the class were paired with the focus students. Therefore, the focus students were not the only ones whose voices were recorded. Consequently, all 23 of the students in my mathematics class received consent/assent forms (see Appendix A). Only the students who returned the forms were paired with the focus students. Although the entire class was instructed together, data were collected only from the 12 students who returned their consent/assent forms.

I am a female teacher currently in my fourth year of teaching elementary school. At the time of this study, I had previously taught a fifth/sixth combination class one year and was in my third year of teaching a sixth grade class in the Utah public school system. I have earned a Utah Level 3 mathematics endorsement as well as an English as a Second Language (ESL) endorsement. During my undergraduate education, I took four courses in mathematics/mathematics education that helped me prepare to use inquiry methods for teaching mathematics. In practice, however, I had previously used inquiry methods only sporadically throughout each school year. This experience was my first time implementing an entire unit using inquiry methods.

Data Sources

As I conducted my study, I collected data from sources strategically designed to help me answer my research question. Both the students and I generated these data.

Student-generated sources. Prior to the unit, I administered a performance assessment to evaluate levels of procedural and conceptual knowledge (Bahr, 2007). The procedural assessment was administered to the whole class (see Appendix B), with each student being asked to complete as much as she or he could. I then evaluated this assessment to determine students' procedural levels. Next I selected a word problem for each student from a bank of problems I had designed to be at varying levels of procedural difficulty (see Appendix C). I distributed these word problems, which all students were expected to complete, but I interviewed only my focus students. Individually they were asked to explain how they solved their word problems (see Appendix C for interview protocol). These interviews were audio recorded and analyzed for conceptual understanding. A parallel performance assessment was given at the end of the unit.

Audio recorders were placed near my four focus students to collect their spoken discourse during each entire lesson in the unit. I made sure that the pairs of students were organized so that the voices of my focus students could be readily distinguished. The audio for the think-pair-shares was transcribed, along with other mathematics-related verbal responses from the focus students. Verbal interludes in which students discussed topics unrelated to mathematics, e.g., PE class and sixth grade romances, were not transcribed. During a think-pair-share, students were given a question first to think about on their own. They were then given time to discuss their thoughts with a partner. Next, each partnership shared with another partnership. Finally, students were given time to volunteer and share their ideas with the class.

Additional data were derived from student written work obtained in the course of typical instruction, thus providing access to the students' written discourse. Written work included (a) the papers on which the students recorded their thinking as they solved their daily mathematics tasks, and (b) fraction logs comprising journal entries, picture dictionaries, and graphic organizers. These data sources also provided a reference to help me interpret the audio recordings. Although a variety of mathematical representations are identified in the literature, (e.g., Lesh, Doerr, & Zawojewski, 2003), I chose to analyze mainly the pictorial representations included in the students' written work.

Teacher-generated source. In addition to transcribing discourse and collecting student written work, I took anecdotal notes. These notes were based on my observations of my teaching as well as on the learning of the focus students during the mathematics lessons. These anecdotal notes then helped me write journal entries for each lesson. This journal was an additional data source.

Design

As I conducted my study, I used an action research approach. I implemented a qualitative design using inductive analysis; rather than using *a priori* codes, codes were generated as the analysis was being conducted. I followed the five essential steps involved in action research, as presented by Johnson (2002):

> First, ask a question, identify a problem, or define an area of exploration. Determine what it is you want to study. Second, decide what data should be collected, how they should be collected, and how often. Third, collect and analyze data. Fourth, describe how your findings can be used and applied. You create your plan for action based on your findings. And finally, report or share your findings and plan for action with others. (p. 13)

By following this action research approach, I was hoping to help my EL students develop strategies for improving their mathematics vocabulary, both for the current topic under study and for future use in learning additional mathematics vocabulary. I was also hoping to understand more clearly processes for teaching mathematics vocabulary to ELs.

Procedures

This thesis completes the steps in the action research cycle that Johnson (2002) deemed necessary. I have asked a question and determined what data should be collected. I studied what happened to the mathematical thinking of ELs as they were taught mathematics vocabulary through research-based strategies. I administered a two-part pre-assessment to assess my students' understanding of both procedural and conceptual knowledge to be encountered in the fractions unit; a parallel post-assessment was given at the end of the unit (see Appendixes B and C). As I taught a 10-lesson inquiry unit on fractions (see Appendix D for lesson plans), I used five research-based strategies to teach mathematics vocabulary to my students: think-pair-shares, journals, graphic organizers, picture dictionaries, and word walls. The journals, graphic organizers, and picture dictionaries were all included in the fraction logs that the students kept. This vocabulary instruction was embedded in the mathematics lessons, rather than being taught separately.

Throughout the unit, I collected and analyzed multiple forms of data to help me answer my research question. I recorded and transcribed the spoken discourse of the focus students. I collected students' logs (which included journal entries, picture dictionaries, and graphic organizers). Assessments and interviews with the students were also analyzed. These data sources revealed the students' representations and discourse and were windows to students' thinking. After I gathered my data, I analyzed them using the qualitative analysis methods described in the next section, and I determined how the information I synthesized can help other teachers and myself in improving mathematics vocabulary instruction. I have completed a major component of the final step, reporting my findings, by finalizing, defending, and submitting my thesis.

Data Analysis

I analyzed the students' spoken and written discourse as well as their pictorial representations. Researcher journal entries, audiotape transcriptions from class (including think-pair-shares), interview transcriptions, and student work samples (including student logs and assessments) were the primary data sources used in this study.

I reviewed the journal, transcriptions, and work samples and segmented them into meaningful units. As I read the text one line at a time, I asked myself the questions recommended by Johnson and Christensen (2008): "Do I see a segment of text that has a specific meaning that might be important for my research study? Is this segment different in some way from the text coming before and after it? Where does this segment start and end?" (p. 534). After segmenting the text from my data sources, I began to code my data. I used inductive coding, which means the codes I used were generated as I examined the data I had collected (Johnson & Christensen, 2008). This process of generating the codes included reviewing the meaningful segments of data and creating codes that described the situation. As I coded the data, new codes were added based on the ideas that emerged.

After coding the data, I looked for relationships among codes (Johnson & Christensen, 2008). I grouped the codes into categories and then examined relationships among the categories. Themes emerged from this examination of categories and helped me organize these categories. These themes included (a) Use of Mathematical Processes, (b) Content Knowledge, (c) Mathematical Accuracy, (d) Mathematical Confidence, (e) Mathematics Vocabulary, and (f) Use of Metacognition. The themes that emerged from my data are validated by their similarity to the five strands of mathematical proficiency described by the National Research Council (2001): (a) Conceptual Understanding, (b) Procedural Fluency, (c) Strategic Competence, (d) Adaptive Reasoning, and (e) Productive Disposition. The strands Conceptual Understanding and Procedural Fluency fit within my theme of Content Knowledge. Strategic Competence is similar to my theme of Mathematical Accuracy, while Adaptive Reasoning is a large part of my Use of Mathematical Processes theme. The strand Productive Disposition is very similar to my theme Mathematical Confidence. While the themes emerged specifically from the spoken discourse of the focus students during the 10 lessons, they were also used in the analysis of the other data sources. The themes helped me make sense of the mathematical thinking of my focus students throughout this fractions unit.

Another individual, an elementary mathematics educator, also coded a portion of the data. We coded the pre-interview, Lesson 1, and Lesson 2 independently of one another. We then examined our coding together to check for consistency. We refined codes that needed adjustments. We then determined that I could continue coding on my own because our coding was consistent, although we did not determine a level of inter-rater reliability.

Because mathematics vocabulary was central to my study, I also used an online vocabulary profiler (Cobb, 1994) to help me identify the mathematics vocabulary used in the students' discourse, and also to help me analyze the kinds of vocabulary words my focus students used. The vocabulary profiler allowed me to compare the mathematics vocabulary usage of my focus students.

Chapter 4

Results

The purpose of this study was to examine the mathematical thinking of ELs as they were taught mathematics vocabulary during a unit on fractions. Research-based instructional strategies for teaching vocabulary were implemented in an elementary mathematics class that was taught using an inquiry approach. The findings of the study described in this chapter emerged from a close analysis of the data collected as described in Chapter 3.

I have organized my results by discussing them in two sections. The focus students' pre- and post-performance assessments are presented first in order to show the students' starting and ending points for this study. Following this section, the results that occurred during the course of the unit are explained in the context of the themes that emerged from the data. I have included children's original spelling and grammar in quoted material to maintain the integrity of their thinking.

Pre- and Post-performance Assessments

Immediately prior to this fractions unit, I gave a performance assessment that evaluated students on both their procedural knowledge and conceptual knowledge. The first part of the assessment had 14 items, with each item evaluating procedural knowledge at a progressively more difficult conceptual level (see Appendix B). The procedural assessment served two purposes. First, the procedural assessment served as a measure of the students' procedural understanding. Second, it was used to find an estimated level of success with mathematics procedures related to this unit in order to select an appropriate level for the conceptual portion of the assessment. Using a method described by Bahr (2007), I used the results from the procedural assessment to help me select an appropriate conceptual word problem for each child (see

Appendix C). I noted the level of complexity of the most difficult item each student was able to answer correctly on the procedural assessment. Therefore, I was able to get an estimate of the level of procedural problems with which each student likely would be successful. I then selected a word problem at one level higher than estimated from the procedural assessment. This word problem was given to the student to solve. If a student answered it incorrectly, he or she was given a word problem one level lower to solve. I used qualitative analysis methods to score the word problems rather than using the rubric described by Bahr (2007). The qualitative analysis methods I used served the purposes of my study. After completing their word problems, I conducted interviews with my four focus students. I asked the students to explain to me how they solved their word problem(s). This process was repeated at the end of the unit as I administered a parallel post-performance assessment. The pre- and post-performance assessments have face validity because the concepts that were assessed were explicitly matched to the concepts that were taught in the unit.

Each of my four focus students improved her or his scores on the procedural assessment over the course of this unit (see Table 1). Laura improved the least. She correctly answered three of 14 on her pretest and six of 14 on her posttest. Yolanda improved the most, with only one of 14 items answered correctly on her pretest and 11 of 14 on her posttest.

Table 1

Number of Correct Answers on Pre- and Post-procedural Assessments According to Focus Student

Assessment	Kyle	Laura	Megan	Yolanda
Pre-assessment score	10	3	3	1
Post-assessment score	14	6	11	11

Kyle's performance. On his procedural pretest, Kyle made a few mistakes simplifying or finding equivalent fractions. For example, he correctly added 2/5 + 1/10 as 5/10, but then simplified his solution as 1/5. However, I could see evidence through his work on other items that he did know how to simplify fractions and find equivalent fractions. He correctly simplified 16/24 to 2/3. He answered 4 of the 5 subtraction items correctly. The item he answered incorrectly involved subtraction of mixed numbers that required regrouping. Looking at the item immediately preceding that one, I could see that Kyle could correctly subtract mixed numbers with uncommon denominators. When selecting his performance assessment word problem, I decided to give him a word problem that was one level more difficult than his estimated procedural level, which was the most difficult level of word problem used in this unit. This word problem involved subtraction of mixed numbers requiring regrouping. The word problem read, "My dog is 5 ½ years old. My cat is 3 ¾ years younger than my dog. How old is my cat?" Kyle answered this word problem correctly (see Figure 1), and therefore did not need to answer a second word problem (only students who incorrectly answered their first word problems were given a second word problem to answer). During the interview, I asked Kyle to explain to me how he solved the problem. He said,

> Um, I knew that 5 and one half was 5 and 5 [5.5] so I subtracted it from this which is 3 and um, 75 tenths [3.75]. When I subtracted it, it equaled 1 and 75 tenths [1.75], so that's how I knew, um, it was, he was one, she was one years old and three and a half, three fourths of a year. (February 26, 2010)

I then asked him how he knew 1.75 was equal to 1 ¾, to which he responded, "Um, `cause, um, because it's right here, or, and because I know, um, fractions that much" (February 26, 2010). Kyle was able to use his understanding of decimals to help him solve this problem. He was able to correctly solve this kind of problem when presented to him in the context of a word problem but was unable to solve a very similar problem when presented in symbolic form. Also, his confusion regarding the mathematics vocabulary of decimals did not appear to hinder his mathematical thinking.

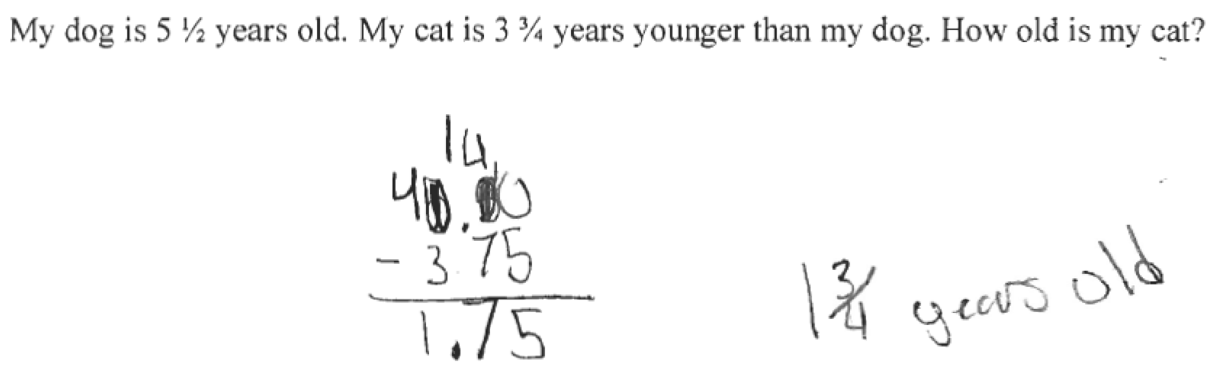

Figure 1. Kyle's answer to pre-interview word problem.

Kyle correctly answered every item on his procedural posttest. He was, once again, given the hardest level of word problem to answer for his interview. The parallel word problem uses the same stem as the word problem Kyle answered during his pre-performance assessment, but with different numbers: "My dog is 7 1/3 years old. My cat is 4 7/9 years younger than my dog. How old is my cat?" This time, Kyle correctly solved the word problem by changing the two mixed numbers into improper fractions with common denominators and then subtracting. He simplified by changing the resulting improper fraction back into a mixed number. By examining both his written work and his oral response during the interview, I observed that Kyle does many steps mentally, without verbalizing or writing them (see Figure 2). He seemed to be able to keep track of the procedure without needing to record every individual step.

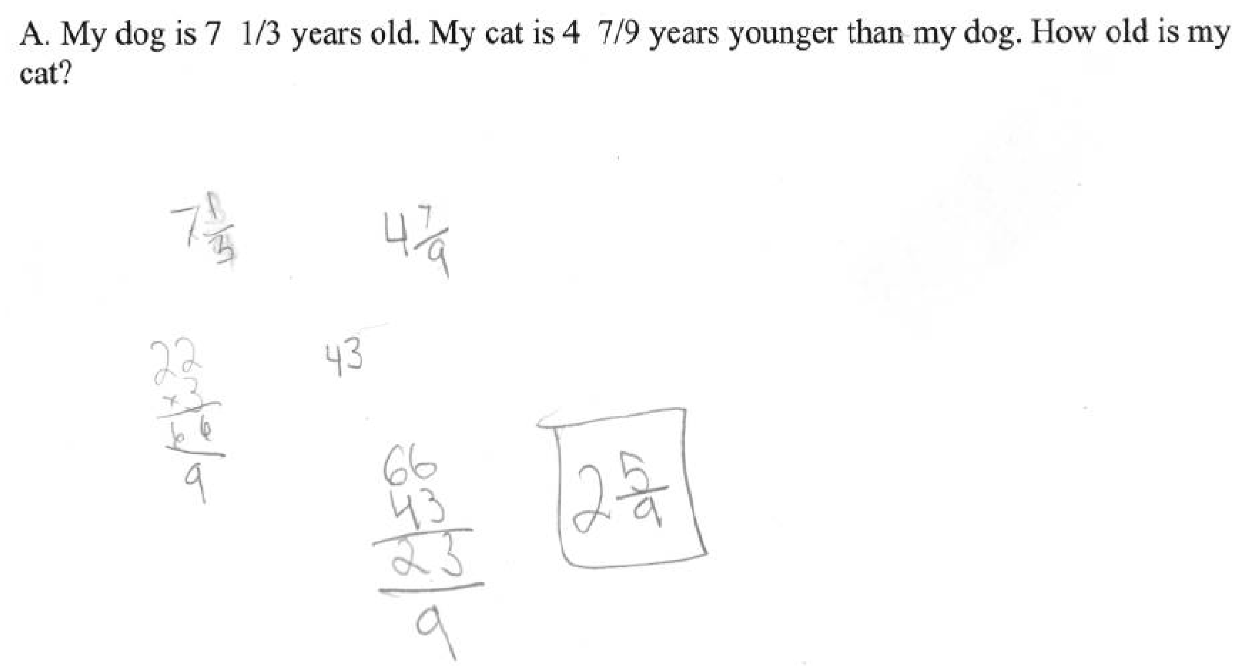

Figure 2. Kyle’s answer to post-interview word problem.

Laura’s performance. On her procedural pretest, Laura answered only three items correctly. She was able to find two fractions equivalent to 1/2, write 4/12 in simplest form, and add 1/7 + 3/7. Because she was able to add two fractions with a common denominator, I gave her a word problem for her interview that was one level more difficult. This word problem involved adding two fractions with different denominators, where one denominator was a factor of the other. The problem she was given read, “My recipe calls for ½ cup white flour and ¼ cup whole-wheat flour. How much flour do I need in total for my recipe?” During the interview, I asked her to explain how she found her answer. She said, “Um, I did half of a circle, then I realized, then I saw one fourth, and then I added, just across the line, and then I added one more” (February 26, 2010). Through this response, Laura showed me that she understood how to represent this fraction through pictures (see Figure 3). She was able to answer the word problem without finding common denominators by using her conceptual knowledge.

I. My recipe calls for ½ cups white flour and ¼ cups whole-wheat flour. How much flour do I need in total for my recipe?

Figure 3. Laura's answer to pre-interview word problem.

Laura was inconsistent with her use of procedures on the procedural posttest. She demonstrated correct procedures such as finding equivalent fractions with common denominators, but only for some of the items requiring such procedures. The only addition item she answered correctly involved adding two fractions with a common denominator. For the other items, she added the numerators and then used the larger of the two denominators as her denominator in the answer. However, she answered three of the five subtraction items correctly, including some that required finding common denominators. The word problem I gave her for her interview involved subtracting fractions with different denominators, where neither denominator was a factor of the other. It read, "My dog is 4/5 year old. My cat is 1/4 year younger than my dog. How old is my cat?" Laura correctly found common denominators and subtracted to find the correct answer: 11/20. When she explained how she found her answer, it was clear to me that she was confused about some mathematics vocabulary, but she understood how to follow the procedure. This was our conversation:

Me: I just want you to explain how you solved this problem.

Laura: Um, I just found the greatest common multiple and then I got five times four and four times five, and then I did it to the top and bottom and I got sixteen twentieths and five twentieths and then it equals to eleven twentieths.

Me: Ok, so what is twenty?

Laura: Umm, the denominator.

Me: Ok, and when you saw this, what denominators did you start with?

Laura: Five and four.

Me: And what was wrong with that? Why'd you have to change them?

Laura: `Cause they were different numbers.

Me: Ok, so you needed to find what?

Laura: The <pause> common fraction. (April 1, 2010)

Although she was confused with vocabulary words such as *greatest common multiple* and *common fraction*, she was able to follow the procedure she had memorized in order to solve the word problem correctly.

Both Kyle and Laura demonstrated the ability to answer a word problem correctly that was at a higher level of difficulty than their demonstrated level of competence on the procedural tests. The context of the word problem may have helped them answer conceptually, even if they did not know a common procedure to answer the word problem. Megan and Yolanda, on the other hand, were not able to answer the more difficult conceptual word problems for both the pre-interview and post-interview.

Megan's performance. During her pre-interview, Megan correctly answered the addition word problem involving fractions with common denominators. This word problem matched her demonstrated level of competence and was given to Megan after she incorrectly answered the word problem that was one level more difficult. The word problem that she answered incorrectly read, “My recipe calls for ½ cup white flour and ¼ cup whole-wheat flour. How much flour do I need in total for my recipe?” She wrote ½ + ¼ = 2/4. When asked to explain how she solved the word problem, she said, “Um, this one, I didn’t get because the denominators were different. So I just added the top ones and put one of these numbers on the bottom” (February 26, 2010). She demonstrated metacognition as she recognized that she had a gap in her knowledge. She knew that she did not know how to solve the problem because the fractions had different denominators. She also demonstrated that she had some prior knowledge regarding vocabulary of fractions through her use of the word *denominators*.

Megan demonstrated much improvement of procedural knowledge over the course of this month-long unit. On her procedural pre-assessment, she was able to add and subtract only fractions with common denominators. On the post-assessment, she was able to answer all of the addition and subtraction items except for the last subtraction item involving mixed numbers requiring regrouping. She left this item blank.

I gave Megan two different word problems to answer for the post-interview. She answered the more difficult word problem incorrectly, but correctly answered the word problem matching her procedural level (see Figure 4). The word problem she answered correctly read, “My dog is 5 5/6 years old. My cat is 3 1/3 years younger than my dog. How old is my cat?” The following exchange contains the dialogue from our interview:

Me: So tell me how you solved this one.

Megan: I saw that the dog was five and five sixths years old, and the cat was three and one third years old.

Me: Years—?

Megan: Younger, years younger than my, than the dog. So I decided to subtract, but then I saw that the denominator wasn't the same. It was half, so I timesed it by two, that would make six, just like the other one. So I wouldn't have to multiply the other one. Then I multiplied the top number by the same thing, two. Then I got three and two sixths. Then I subtracted the big number, and then the top number, and I just left the denominator the same. And then I simplified it too, by three.

Me: Ok, and what was your answer?

Megan: Two and one half. (April 1, 2010)

This conversation demonstrates considerable improvement in Megan's procedural knowledge over her initial knowledge displayed at the beginning of this unit. She also used mathematics vocabulary more frequently during her post-interview than she did in her pre-interview.

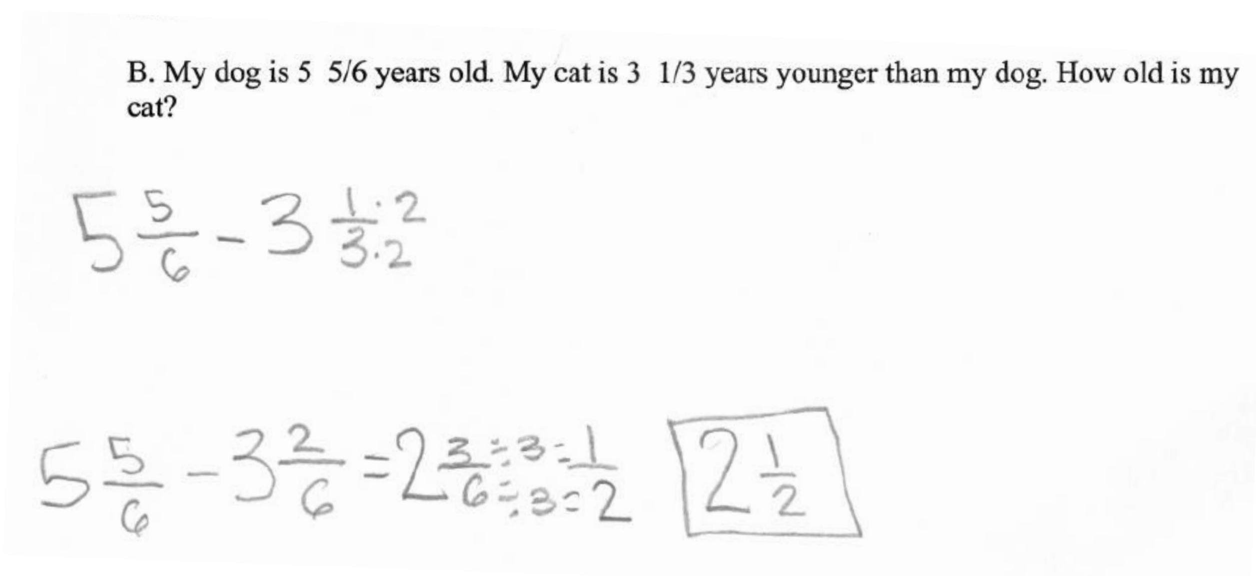

Figure 4. Megan's answer to post-interview word problem 1.

Megan's work for the more difficult word problem showed that she had attempted to memorize the procedure for subtracting mixed numbers with regrouping, but it is clear that she does not have conceptual understanding of the procedure (see Figure 5). She followed some steps correctly but made an error when regrouping. When she wrote four and nine ninths she was attempting to regroup the wrong number. She knew to regroup one of the wholes into a fraction with the same denominator, but she did not subtract that whole number from the first mixed number. Her last step should have been six and twelve ninths minus four and seven ninths, but because she did not subtract one of the wholes from the original whole number, her final answer ended up being one whole number too high—three and five ninths rather than the correct answer, two and five ninths. Her inability to see her error demonstrated a gap in her conceptual knowledge.

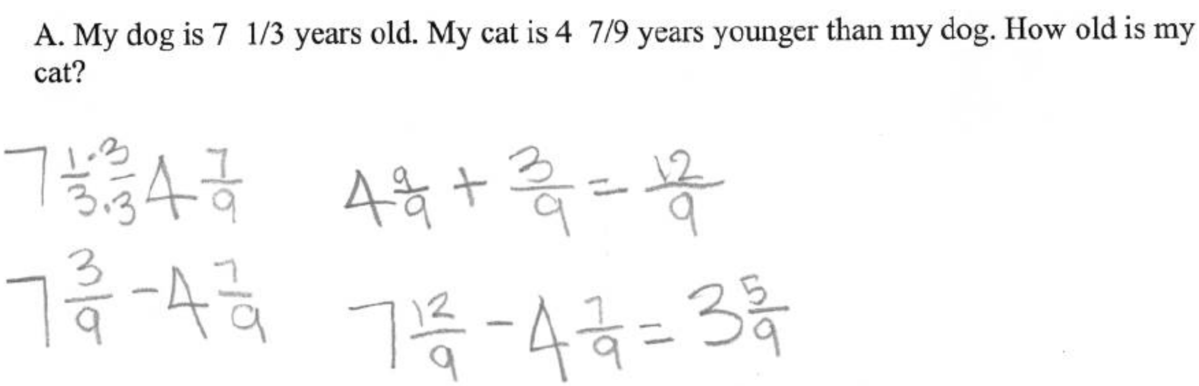

Figure 5. Megan's answer to post-interview word problem 2.

Yolanda's performance. Yolanda, too, showed considerable improvement in procedural knowledge during this unit. She received credit for only one answer on her procedural pretest, and even that correct answer demonstrated a gap in knowledge (see Figure 6). The item asked for two drawings that were equivalent to a given drawing depicting 3/4. Yolanda drew two different representations of the same fraction, 3/4. She answered every item requiring adding, subtracting, and simplifying incorrectly. When asked to simplify the fraction four twelfths, Yolanda wrote three twelfths (see Figure 6). Her answers on her procedural pretest demonstrated very little fraction knowledge. On her posttest, however, she answered every addition and subtraction item correctly, only making errors on three items involving simplifying and equivalence.

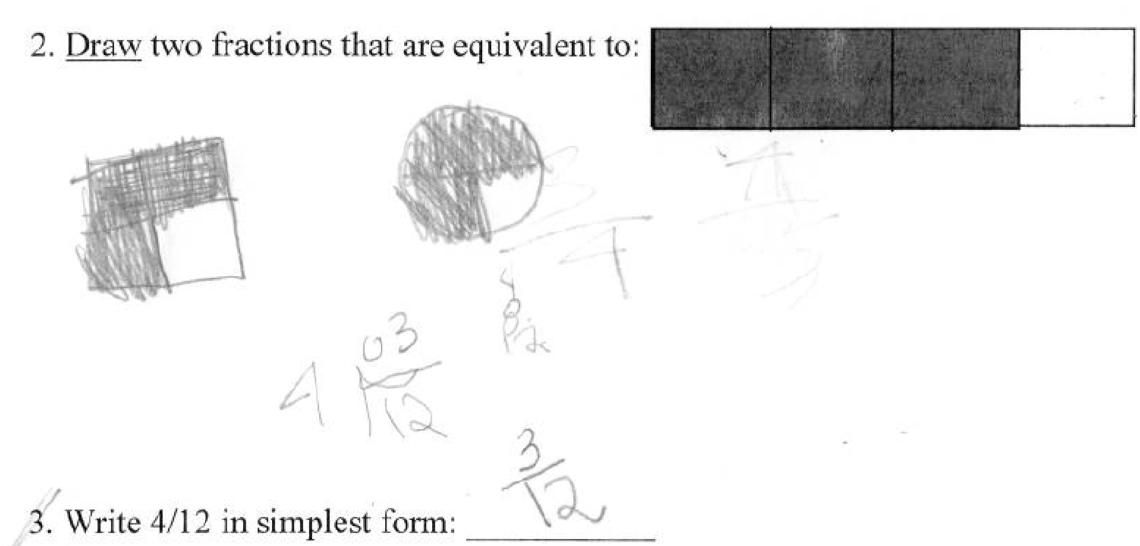

Figure 6. Sample items from Yolanda’s procedural pretest.

During her pre-interview, Yolanda explained how she solved the word problem, “My recipe calls for 1/8 cup white flour and 5/8 cup whole-wheat flour. How much flour do I need in total for my recipe?” She said, “I just lined them up like adding, and then eight plus eight equals 16, and one plus five equals six” (February 26, 2010). In this explanation, Yolanda demonstrated a lack of fraction knowledge as she added both the numerators and the denominators. After answering this word problem incorrectly, she was given an easier word problem that more closely aligned with her demonstrated ability. The word problem read, “I made a pan of brownies and my brother ate 4/12 of them. Show me a different way to write the fraction of brownies he ate. (You may draw pictures.)” She drew a rectangle and drew vertical lines to split the rectangle into 12 pieces. She then shaded in four of them. This representation demonstrated that she understood what the four and 12 in the fraction meant. During the interview, I prompted her to answer the question asked in the word problem by asking if there was another way to write that fraction. The following conversation then took place:

Yolanda: Um, yeah, there is. Like which way, like different way, like 12 out of four?

Me: Well, like maybe different numbers that mean the same thing.

Yolanda: I think, um. <long pause>

Me: If you're not sure, that's ok.

Yolanda: I think it's going to be two. Two. Two over, six?

Me: Ok, and how did you find that out?

Yolanda: `Cause four is half of two, and twelve is half of six. (February 26, 2010)

During this interview, Yolanda demonstrated confusion over basic fraction concepts. With extensive prompting, she was able to state a fraction equivalent to 4/12; however, her explanation included reverse relationships (e.g., four is half of two), and signifies a gap in understanding.

Because of her considerable improvement on her procedural posttest, Yolanda was given much more difficult word problems to answer during the post-interview. She answered the most difficult word problem available incorrectly, but the next-most difficult word problem correctly. She was unable to subtract mixed numbers with regrouping correctly in the context of a word problem, even though she solved a similar problem correctly on the procedural posttest. The work she showed as she attempted to solve the word problem demonstrated partial understanding of the procedure but a considerable lack in conceptual knowledge. Because she answered the first word problem incorrectly, she was given an easier word problem to answer. On this word problem, Yolanda was able to find fractions with common denominators and subtract the mixed numbers correctly.

Summary of performance assessment results. All of the focus students made improvements on both procedural and conceptual knowledge over the course of this unit. These improvements are evident through examination of the procedural pre- and post-tests and the pre- and post-interviews. Megan and Yolanda both made substantial improvement. Kyle began the unit at a higher level than the other three focus students, and because the assessment had a built-in ceiling, could not demonstrate the same increase in scores from his procedural pretest to the procedural posttest. He did, however, get a perfect score on the procedural posttest and was able to answer his post-interview word problem clearly and accurately. Laura is the only focus student who demonstrated little growth throughout the unit.

After examining the focus students' level of knowledge prior to the unit and comparing that knowledge to their knowledge at the end of the unit, we can clearly see that substantial growth and improvement took place for these students during this unit. The following sections describe what happened to the students' mathematical thinking during the fractions unit that may have contributed to the students' progress.

Use of Mathematical Processes

After administering the performance pre-assessment, I taught 10 lessons on fractions to my sixth grade mathematics class. Research-based vocabulary strategies were embedded into the mathematics lessons. I audio-recorded the four focus students, Kyle, Laura, Megan, and Yolanda, during each lesson. I then listened to the recordings and transcribed the students' words each time they engaged in spoken discourse about mathematics. After transcribing each recording, I generated codes according to the data that emerged. As I coded the spoken discourse in the 10 lessons, I noticed that some of the codes were related to each other. These codes grouped naturally into categories. Next, I looked at how the categories were related. Many categories seemed to fall under common themes (see Appendix E). One of these themes is Use of Mathematical Processes. The categories that fall under this theme are Communication, Justification, Representations, and Connections.

Communication. My focus students communicated both orally and in written form throughout this unit on fractions. Students' oral communication was audio-recorded and transcribed. Students' written communication was also collected for analysis.

Oral. Spoken discourse is a major means of communication in an inquiry-based classroom. Through the transcription of my focus students' spoken discourse, a substantial amount of data was collected. The number of words spoken throughout the unit differed from student to student. Some students talk more than others, and these differences were especially evident among my focus students when I focused on their conversations about mathematics. Students' discourse was transcribed only if related to mathematics; discussions about students' social lives were not transcribed. Therefore, the focus students may have spoken more than is reflected in the transcriptions, but only mathematics-related speech is reported here. The number of words, or tokens, transcribed for each focus student was counted for all 10 lessons. The word *token* refers to the total number of words spoken, including repeated use of the same words. The large difference in tokens among the focus students is evident in Table 2.

Clearly, Kyle spoke about mathematics using many more tokens than did the other three focus students. In fact, he used almost double the number of tokens the student with the next-highest word count used, and nearly four times the number of tokens spoken by the student with the fewest tokens. It is important to keep these differences in mind as we look at the frequency of codes for each student. Because Kyle had spoken so many more tokens in mathematics-related conversations than did each of the other three focus students, it would appear likely that Kyle would have more codes emerge from his data. After calculating the frequency of codes for each focus student, Kyle did in fact have more occurrences of codes than the other three focus students.

Table 2

Number of Tokens Spoken by Lesson According to Focus Student

Lesson	**Kyle**	**Laura**	**Megan**	**Yolanda**
1	208	104	52	79
2	320	295	378	60
3	307	75	87	20
4	385	39	61	46
5	-	46	193	-
6	185	25	-	87
7	46	20	57	70
8	101	116	16	27
9	49	88	23	26
10	132	35	25	52
Total	1,733	843	892	467

Note. Kyle was absent for Lesson 5 and Megan was absent for Lesson 6. Yolanda did not speak about mathematics at all during Lesson 5.

On the day I taught Lesson 5, the two other members in Yolanda's group were absent. Because Kyle was also absent for Lesson 5, I decided to have Yolanda work with Kyle's group. The two other members of Kyle's group appeared not to offer the support that Yolanda's group members typically offered her. Yolanda and the two other members of this group did not converse much during Lesson 5. In fact, Yolanda did not talk about mathematics at all during the entire class period. The following is an excerpt from the journal entry I wrote on March 17, the day I taught Lesson 5:

> Yolanda managed to go the whole class period without saying anything about mathematics loud enough to be recorded. This might have partly been due to the new group dynamics. Anna is usually so good about helping Yolanda along. Phillip and Jill would both prefer to work on their own.

The names Anna, Phillip, and Jill are pseudonyms for other students in my class. Anna was in Yolanda's group, but was absent for this lesson. Phillip and Jill were in Kyle's group, and they did not interact much with Yolanda.

Think alouds. One aspect of oral communication in mathematics is thinking aloud while working to solve a mathematics problem. Kyle was recorded thinking aloud 27 times during the course of the 10 lessons. The number of think-aloud occurrences for Kyle seems high when compared to Laura and Megan, who thought aloud eight and nine times, respectively, and Yolanda who was only recorded thinking aloud one time (see Table 3).

Instruction during Lesson 2 (lesson plan included in Appendix D) seemed to have encouraged thinking aloud for Kyle, Laura, and Megan. In this lesson, Kyle thought aloud as he used Cuisenaire rods (rods that are color-coded according to length) to help him show that two thirds is equivalent to four sixths. The following demonstrates his thinking: "So six. How many?

Sixths. One third. So there's four of them. And there's two of them. Oh, this is getting easier! I think it's a green one. Yes!" (March 4, 2010). In the same lesson, Megan thought aloud as she searched for relationships among the Cuisenaire rods: "Let's see, equal, one brown one equals two purples. One whole, equals, it doesn't fit! Two white equal one red. One red" (March 4, 2010).

Table 3

Number of Occurrences of Communication According to Focus Student

Type of Communication	Kyle	Laura	Megan	Yolanda
Number of think-alouds	27	8	9	1
Number of student-generated questions	13	10	9	13

The number of think-alouds seemed to correspond loosely with the total number of words spoken throughout the 10 lessons. In other words, Kyle had the most recorded instances of thinking aloud, with Laura and Megan having many fewer recorded instances of thinking aloud than did Kyle; Laura and Megan were recorded thinking aloud about the same number of times. Yolanda, who spoke many fewer tokens than did the other three, also used considerably fewer think-alouds.

Student-generated questions. Another important facet of communication is student-generated questions. All four focus students asked about the same number of questions during the 10 lessons, despite the fact that Kyle typically used more tokens than did the other three (see Table 2). Kyle and Yolanda both asked 13 questions, while Laura asked 10 and Megan asked nine. Although they asked questions about the same number of times, the kinds of questions the focus students asked varied greatly. Most of Kyle's questions were clarifying questions; he repeated what I previously said, but in question form. He also asked if he had found the correct answer and how to spell the words *triangle* and *hexagon*. Occasionally, Kyle asked about the assignments, as seen in the questions, "Do we have to do this one on the other side?" (March 4, 2010) and "So we just share all of it?" (March 4, 2010).

The questions of the other three focus students were more closely related to the content. Laura and Megan asked questions such as, "Is this equal?" (March 4, 2010) and "What's one third?" (March 4, 2010). Similarly, Yolanda asked, "Would it be three?" (March 4, 2010). These questions signify confusion about the mathematics content. Megan and Laura asked numerous questions associated with which operation they should use to solve a problem. Megan's questions of this kind were usually clarifying questions, such as, "It's subtraction on all of them, right?" (March 26, 2010). Laura, on the other hand, often asked with less surety, "Do you add or subtract?" (March 31, 2010). Yolanda asked a question regarding which operation to use once during the 10 lessons.

Many of Yolanda's questions were related to mathematics vocabulary. During Lesson 1, she asked several questions about the pattern blocks, such as, "Are these called a hexagon?" (March 2, 2010) and "What's a rhombus again?" (March 2, 2010). Yolanda showed a desire to gain deeper understanding through her questions: "But how'd you get five and three fourths?" (March 24, 2010) and "Can you explain it to me?" (March 24, 2010).

Think-pair-shares. Think-pair-shares were conducted in six of the lessons, and the audio from these discussions was recorded and transcribed. Because of the limited number of consent/assent forms that were returned, I formed collaborative groups of three rather than four students. This grouping was not conducive to traditional think-pair-shares in which pairs of students first share their ideas with each other and then with another pair. Typically, the three students in a group shared their ideas with each other during the *pair* portion of the activity, and then two groups of three would share with each other during the *share* portion. Sometimes the focus students appeared to ponder the questions in the think-pair-share activities. Other times, however, the students did not discuss the questions. In Lesson 1, students explored fractional relationships among pattern blocks, which consist of triangles, rhombuses, trapezoids, and hexagons that are proportionally related. The question I asked was, "What relationships did you notice between the different pattern blocks?' Kyle said, "It takes two, it takes two just to make one, three. I'm gonna write this thing in here. We'll start from little to biggest. How do you spell triangle?" (March 2, 2010). Kyle's two partners then shared their ideas, both sharing responses that clearly answered the question. Clearly, Kyle attempted to share his ideas, but he did not actually answer the question, or even share a complete idea. He decided to write in his log rather than finish his think-pair-share ideas. Laura's response was, "Three triangles equal a trapezoid. I think these are trapezoids" (March 2, 2010). This comment started a conversation between Laura and her partner, with each of them sharing different relationships they noticed. Megan and Yolanda's groups both began answering the question but did not persevere.

In Lesson 2, I asked a question similar to the one asked in Lesson 1, but this one was in regard to the relationships found with the Cuisenaire rods. Kyle asked, "So we just share all of it?" I told him to share some main ideas, to which he responded,

> Uh, the bottom ones, uh, they're like, you multiply them by two, like top one by 2, then you get two sixths. Yeah, you should do the biggest to smallest. Then it'll equal out like this. See, look, it'll equal out like this. (March 4, 2010)

Kyle used imprecise words (e.g., "bottom ones") to explain his ideas about the fraction relationships he noticed. He also shared several incomplete thoughts. Deciphering what he meant by this verbal response is difficult.

Megan shared incomplete thoughts as she answered the think-pair-share questions in Lesson 2; however, her responses demonstrated some understanding of fraction relationships. In response to the first question, which asked students what they noticed about the lengths of the rods, she said:

> Purple one. Two red ones. Remember how purple one, two reds, and then four little ones are the same thing, or whatever? And then one half and two fourths is. Umm, two reds equal one purple one and four, four white ones equal a purple one. And if I took one of the red ones off, it would be one half, and then if I took two of the white ones off it would be two fourths.

For the second think-pair-share questions, which asked specifically about the relationships among the rods, she stated, "Umm, three greens equals one blue, and nine whites equals three greens. And two sixths and three ninths are equivalent fractions" (March 4, 2010). Megan not only demonstrated correct usage of mathematics vocabulary, but also connected the relationships between the rods with fraction equivalences.

Yolanda and her partners responded with silence to the think-pair-shares in Lesson 2. During the two think-pair-shares in Lesson 3, Yolanda remained silent while her partners shared their ideas. The first think-pair-share question I asked in Lesson 3 was, "When you were trying to compare the different-sized pieces, like the different sized cookies, what problems did you face?" Kyle responded unclearly by saying, "Um, the problem I faced was, remember this one? Who ate more? Then until now" (March 9, 2010). Kyle appeared to be satisfied with his response, feeling he had fully answered the question. Laura responded to the first think-pair-share question by asking what the question was. Megan, on the other hand, did hear the first question and responded with, "When we not. It actually made sense" (March 9, 2010). This incomplete thought gives little evidence as to what Megan was thinking. The second think-pair-share question I asked during Lesson 3 was a follow-up question to the first one. I asked, "What were some things you had to do in order to be able to compare the different pieces?" Both Kyle and his partner responded by saying they did not know what the question was. There were no responses from Laura or Megan to the second think-pair-share question in Lesson 3.

In Lesson 4, I asked the following think-pair-share question: "I want you to think about the two questions on this page. The first question was, 'How much pizza was eaten?' The next question was, 'Is there more than one way to write the amount of pizza that was eaten?'" I then asked the students to think on their own before sharing. Kyle stated, "The relationship is all on my paper" (March 15, 2010). He did not share beyond that response. Laura said, "Umm, I came up with three and four sixths. Mmm, I came up with an improper fraction" (March 15, 2010). Laura recognized that the amount of pizza could be represented by a mixed number and an improper fraction. She also used a vocabulary word correctly. Neither Megan nor her partners shared any responses to this question. Yolanda, on the other hand, shared a concise and accurate response as she readily answered, "Three and two thirds is equivalent to twenty-two sixths" (March 15, 2010). The students were able to practice using mathematics vocabulary during these think-pair-share conversations.

In order to encourage my students to focus and give thought to the think-pair-share questions, I asked them to write down their thoughts to the think-pair-share questions in Lesson 6 and Lesson 9. This adjustment to the way I implemented think-pair-shares ensured that each student would be able to think through his or her ideas and be prepared with a response. These responses are discussed in the Written Communication section.

Written. Students communicated through writing when they recorded journal entries in their fraction logs. Although students were permitted to write and reflect on their learning in their fraction logs as often as they wanted, many students wrote only when responding to a specific question I asked. After students explored during Lesson 5, I asked, "Why do the denominators stay the same when adding and subtracting fractions?" I then gave the students time to respond in their fraction logs. Laura wrote, "Because that if you add and subtracting then it allways has to be the same if you do 5/8 – 2/8 it will always be 3/8. The denominator always stays the same. So does subtracting the denominator always stay the same when it comes too fractions!" (March 17, 2010). This response demonstrated only surface knowledge. She may have understood a bit of the procedure involved in adding and subtracting fractions with common denominators, but she did not express deeper understanding. The other two girls, on the other hand, demonstrated slightly more conceptual understanding, though neither response was very clear. Megan wrote, "It stays the same because if you were to chang the denominator it would be a diffrent amount. Just like if you had a pan of Brownies and there was 17 brownies you can't Just change it or it will be a diffrent amount of Brownies" (March 17, 2010). She recognized that the denominator is an important part of the fraction and cannot be changed arbitrarily. Yolanda also hinted at conceptual knowledge in her written response, which says, "Because the denominator is how much there is and that why it stays the same" (March 17, 2010).

During Lesson 6, I asked the think-pair-share question, "Why are the words 'common denominator' and 'least common denominator' so important in today's activity?" Kyle wrote, “It was so important because it help us add and sudtrat. It help us becuse it would be hard to add 3/6 + 4/9 and the awner you got is probably wrong” (March 22, 2010). Responding to the same question, Laura wrote, “Because common denominator is the same denominator and when the bottom number stays the same. Least common denominator is when you find the Least common Multiply and it important because these questions are” (March 22, 2010). Yolanda wrote, “Common denominator and least common denominator are so important today because they help us make a common fraction and we can finally find or work out the fraction. If you don’t use a common denominator or a lest common denominator It wouldn’t be right” (March 22, 2010). All three of these responses illustrated surface knowledge that was not very well developed. None of the three expressed understanding of how finding common denominators allows us to add or subtract because we would be dealing with pieces of equal size.

To launch Lesson 9, I wrote a word problem on the board with two blanks where numbers should go: “Sharon made ______ pies. Her friend ate ______ pies. How many pies are left?” The students immediately recognized the word problem as a subtraction problem. I filled in the blanks with different pairs of numbers to create several word problems that progressively became more difficult. My last pair of numbers included mixed numbers. The fraction in the first mixed number was smaller than the fraction in the second mixed number (7 1/5 – 3 3/5). I then asked the students to write in their logs why this word problem was more difficult than the others. Megan responded with the answer that showed the most understanding, albeit surface understanding. She wrote, “It is more difficult because the one is smaller than the three. And if it is subtracting the one has to be bigger than the three to be able to subtract it” (March 29, 2010). Laura, on the other hand, attempted a justification by writing: “7 1/5 and 3 3/5 are hard because you will have to see what goes into 1/5 and 3/5 and plus it’s much more bigger” (March 29, 2010). While concluding what exactly Laura was thinking when she wrote this response is difficult, she appears to be referring to finding common denominators. Kyle wrote, “It’s not, it’s easy because it is easy to do and it’s in simplest form so you can sudtact” (March 29, 2010). He appeared confident and I considered that perhaps he had learned this concept previously. However, when I checked his work, he had subtracted the whole numbers first and then subtracted the smaller fraction from the larger one. He did not recognize that regrouping was necessary. This question became a think-pair-share question as I asked the students to share what they had written.

Overall, I observed that the quality of explanations improved when students were required to write their ideas down before sharing them. This improvement was evident when I compared the think-pair-share responses that were shared only verbally with those that were written first. When students were required to put their thoughts in writing, they made them more coherent than when they did not write them. Another outcome of having students write down their responses first, however, was that the students did not continue talking after they had shared their prepared statement. The students read their written responses and then stopped talking. This adjustment to the think-pair-shares did not encourage discussion of ideas.

Justification. As I began to code the students’ responses to questions, I saw the need to go beyond coding and actually assign scores to the types of justifications students gave. For simplicity and ease of understanding, these scores were assigned codes and were included in my list of codes (see Appendix E); however, they carried the weight of scores, with some answers being considered more desirable than others. Therefore, while many of the data in this study are qualitative, there are some quantitative characteristics involved in scoring the students’ justifications. Three separate situations emerged regarding justification. First, students either answered questions correctly or incorrectly. If they answered correctly, they sometimes gave clear, complete justifications, while other times their justifications were weak or unclear. These two situations were given separate codes. A third code was given in instances when a student attempted justifying an incorrect answer. The ideal situation was for students to give a clear, complete justification for a correct answer.

During the 10 lessons, Kyle attempted justifying 24 answers, 14 of which were clear and complete. The remaining 10 justifications were confusing and difficult to follow (see Table 4). In Lesson 3, the following word problem was asked:

> Emily and Kevin each had the same amount of cookie dough. Emily made 9 small cookies and ate 5 of them. Kevin made 3 larger cookies and ate 2 of them. What fraction of the cookies did each child eat? Who ate more? How do you know?

I later refined the word problem to specify that Emily and Kevin used all of their cookie dough to make their cookies. After drawing pictures (see Figure 7), Kyle justified his answer to this word problem by saying:

> If we divide these in threes. Yeah, Kevin did eat more because if there's nine… If there's nine small cookies, and then just divide them in three, then you see how much there is, like Kevin ate, if you divide this into three Kevin ate like six of them and she only ate five. (March 9, 2010)

While Kyle may have stumbled over his words a bit, his response was mathematically sound and easy to follow. This justification was considered complete.

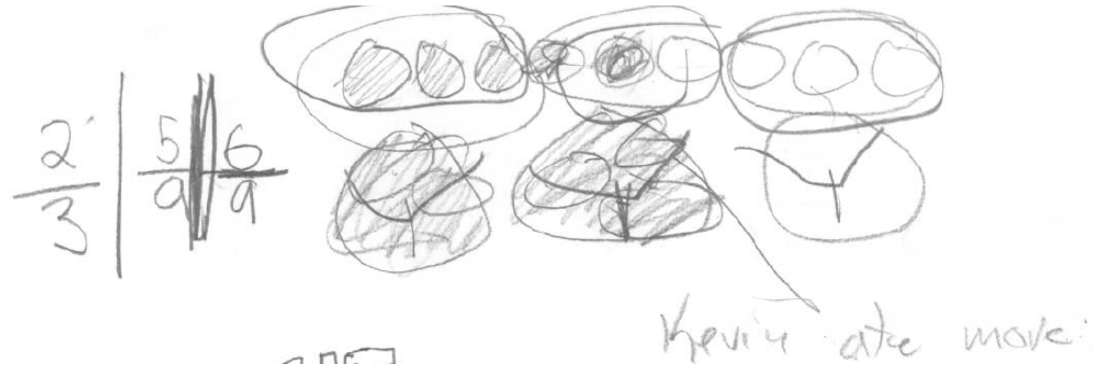

Figure 7. Kyle's representation for cookie problem.

In Lesson 4 Kyle was asked to share what three and four sixths, three and two thirds, twenty-two sixths, and eleven thirds all have in common. He responded, "Oh! On the bottom it's a six, and these are more higher. On the bottom it's a three. They're kinda the same, we put `em in thirds" (March 15, 2010). This is an example of a weak justification. It is confusing and incomplete. Whether Kyle understood the relationships between mixed numbers and improper fractions is unclear. He also lacked a strong explanation for why or how he simplified the fractions.

Table 4

Kinds of Answers and Justifications According to Focus Student

Kind of Justification	Kyle	Laura	Megan	Yolanda
Correct answer				
Clear, complete justification	14	2	6	1
Weak, unclear justification	10	6	4	7
Incorrect answer with attempted justification	0	1	1	2

Note. This table shows the kinds of justifications for only those answers that students attempted to justify. The focus students answered many questions in addition to the ones listed in this table, but did not attempt to justify those answers.

Laura and Yolanda both had at least one justification that fell into each of the 3 categories. Both of them gave many correct answers, but with weak, unclear justifications more often than complete and clear justifications. They had difficulty expressing their reasoning clearly. When Yolanda was asked in Lesson 6 why she did not add the denominators, she responded, “Because it’s how much there is” (March 22, 2010). In this phrase she hints at understanding the concept that the denominator of a fraction tells us how many pieces the whole is broken into, but her response is unclear and incomplete. In response to the following question, also asked in Lesson 6, “Why are the words *common denominators* and *least common denominator* so important in today’s activity?” Yolanda responded,

> Common denominator and least common denominator are so important today because they help us make a common fraction. And we can finally work out the fraction. If you don’t use common denominator or least common denominator, it wouldn’t be right. (March 22, 2010)

In this example, Yolanda demonstrates her understanding that in order to “work out the fraction,” or perform certain computations with fractions, the fractions must have common denominators. However, out of context, her response would make little sense and does not clearly explain how common denominators or least common denominators help us in performing computations. This response lacks clarity and completeness.

Another example of Yolanda’s weak justification can be seen in an exchange from Lesson 3. She was exploring the following word problem:

> Sarah had a pan of brownies that was cut into 8 equal pieces. She gave away 5 of them. Joyce had a pan of brownies that was the same size as Sarah’s, but Joyce’s pan of

brownies was cut into 12 pieces. She gave away 7 of them. What fraction of the brownies did each girl give away? Who gave away more? How do you know?

Yolanda had drawn two horizontal rectangles of equal size, with a horizontal line cutting each of them in half. For Sarah's pan, she added three more vertical lines to make eight pieces, and for Joyce's pan she added five vertical lines to make 12 pieces. Because Sarah ate five pieces, Yolanda shaded in the four pieces on the bottom row, plus one piece on the top row. For Joyce's pan, she shaded in the six pieces on the bottom row and one more piece on the top row. Both pans had the bottom half of the pan shaded in, plus one more piece. The following conversation took place after she had drawn these pictures.

Me: How do those extra pieces compare in size? Which one's bigger?

Yolanda: That one (pointing to the brownie in the top row in the pan of eight brownies).

Me: How do you know?

Yolanda: Because it looks bigger. (March 9, 2010)

This justification demonstrates a lack of fraction knowledge; at the same time it also shows weak justification skills. Yolanda used a similar justification for a question regarding relationships among pattern blocks in Lesson 1:

Me: What fraction of the rhombus is the triangle?

Yolanda: Half.

Me: Half, how do you know that?

Yolanda: Looks half. (March 2, 2010)

In this example, Yolanda relies on her visualization of *half*, rather than an understanding of the fraction one half. While the visualization and the mathematical use are the same in this situation, it is unclear whether Yolanda understands why the triangle is half of the rhombus. Her justification does not provide enough evidence that she knows what a denominator is and what fractions signify.

Laura's justifications were also typically weak. In Lesson 5, Laura correctly answered the word problem, "Julie made a pan of brownies and cut them into 15 pieces. She ate 6 of them that day. The next day, she ate 4 more. What fraction of the whole pan did Julie eat?" When asked why the denominator had to stay the same when adding, she simply stated, "Because there was 15" (March 17, 2010). She seemed to understand that the denominator must stay the same rather than be added; however, her response does not give us a clear picture of her understanding of denominators. Her justification lacked the depth that would provide stronger evidence of her conceptual understanding.

Megan justified 11 responses, with six being correct and clear and four being correct but unclear. There was one instance in which she attempted to justify an incorrect answer. Megan often demonstrated a deeper conceptual understanding in her justifications than did the other two girls. The following exchange in Lesson 5 is dealing with the same word problem about brownies that was previously explored in relation to Laura's response, but a different level of understanding is portrayed in Megan's justification:

Megan: Um, I added six plus four and then I got ten, and I learned that the denominators stay the same.

Me: Ok, how do you know that? Why do they stay the same?

Megan: Because if you change it, it would be a different kind of number of brownies. Like if you change it to five, it would be five brownies. (March 17, 2010)

Because Megan gave an example of using a different denominator, we get a better view of her understanding of denominators. It is clear that she understands the function of the denominator, in this case, telling us the number of brownies in the pan. Even though Megan displayed deep conceptual knowledge occasionally, she often did not have clear, complete justifications. When asked to defend her answer in Lesson 3 that Kevin ate more cookies, she responded, "Because two cookies, he ate two *big* cookies, and she ate five *little* cookies" (March 9, 2010). She appeared to think that because Kevin's cookies were *big* and Emily's cookies were *little*, she had enough information to justify her answer. While she was correct that Kevin did eat more, her justification lacked mathematical reasoning to defend her answer.

Megan occasionally avoided giving justifications to answers. In Lesson 2, I asked Megan to tell me how she knew an answer, to which she responded, "I don't know, my brain?" (March 4, 2010). Similarly, in Lesson 5, when asked how she knew an answer, she said, "Umm, I subtracted from four. I don't know, I just guessed" (March 17, 2010). These responses suggest Megan lacked confidence in her mathematical understanding.

All four of the students had several justifications that were incomplete or difficult to follow. Laura and Yolanda had the fewest complete justifications. Of the total number of justifications Kyle attempted, 58% of them were clear and complete. Nearly 55% of Megan's justifications were clear and complete. Contrast these percentages with Laura's 22% and Yolanda's 10% of justifications being clear and complete. Despite these low percentages, the students often demonstrated correct use of mathematics vocabulary as they attempted to justify their answers.

Representations. I strongly encouraged my students to draw pictures as they explored the fraction ideas in order to help them develop a deeper conceptual understanding. As I transcribed the data, I listened for student references to drawing pictures. Kyle and Laura talked about drawing pictures only once each, with Megan talking about drawing pictures twice, and Yolanda three times. While it may appear that the students did not draw many pictures, I looked through their work samples and found that they had, in fact, drawn many more pictures than they talked about. Kyle had drawn pictures for six of the 10 lessons, and Megan and Yolanda had each drawn pictures for seven of the 10 lessons, while Laura drew pictures for eight of the 10 lessons.

I asked the students to keep a picture dictionary in their fraction logs. Students were instructed to use pictorial representations to help them understand vocabulary words. Despite my explicit instructions that students were to draw pictures to represent each word on our word wall, some students defined the vocabulary words without pictures. Kyle's picture dictionary is largely lacking in pictures. He drew a picture for only the word *equivalent*. He drew two bars: one bar with one of four squares shaded in, and the other with two of eight squares shaded in (see Figure 8). Each of the other vocabulary words only showed numbers, with an occasional arrow pointing to something. He demonstrated surface understanding of the vocabulary words as he pointed an arrow at the top number for the word *numerator,* or two arrows to the denominators in the fractions 9/10 and 2/10 to show *common denominators*. His picture dictionary did not demonstrate his conceptual knowledge.

Laura's picture dictionary does not have any pictures. However, she did try to explain the words in addition to showing examples. Her explanations only demonstrated surface knowledge of the words. She defined the word *equivalent* as "are the same." For *improper fractions* she wrote, "The numerator is bigger than the denominator."

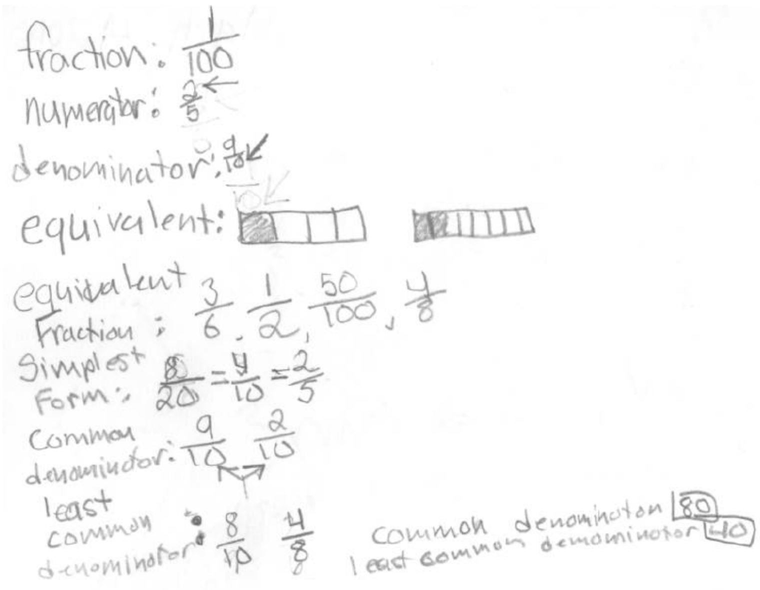

Figure 8. Kyle’s picture dictionary.

Megan actually attempted to draw pictures for her picture dictionary (see Figure 9). She even used colored markers. Her entries hint at a deeper conceptual understanding; however, because she spent so much time making her pictures artistic, she did not have enough time to create entries for all the vocabulary words. For the word *fraction*, she has three different representations for the fraction 4/7. She drew seven cookies, with four being set apart from the rest, and two different versions of four slices of pizza cut into seven slices. For *common denominator* she shows 7/4 + 4/4, but her picture shows seven gifts plus four gifts. From this drawing, I have difficulty knowing what she really understands about denominators. Her entry for *denominator* shows only surface knowledge: “is the botom number.”

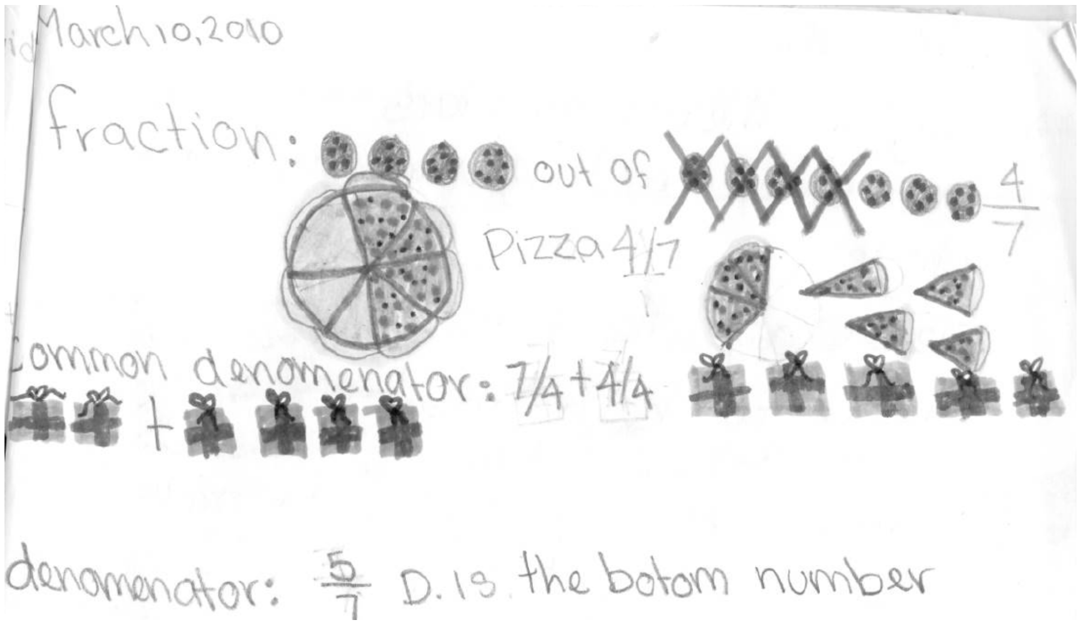

Figure 9. Megan's picture dictionary.

Connections. One of the connections I looked for was a spoken connection to something the students had written or drawn on their papers. Kyle made this kind of connection most often, with 17 such connections. Laura made this connection 10 times, while Megan and Yolanda each made this connection four times. Another connection I looked for was a connection to previous knowledge. The students did not make oral connections between current knowledge and previous knowledge often, but all of the focus students did make this connection except for Yolanda.

In an effort to encourage students to make connections between ideas, I wanted them to create graphic organizers. I had my students create only one graphic organizer, a Venn diagram comparing the words *mixed number* and *improper fractions*. The students appeared rushed when completing the diagram, and many did not quite understand how to complete it.

Kyle's Venn diagram mostly demonstrated surface knowledge. For *mixed number* he wrote, "Whole number and a fraction." For *improper fraction* he wrote, "Top number bigger than bottom." In the overlapping space he wrote, "They can be the same much." Although Kyle has demonstrated deep conceptual understanding previously throughout this fraction unit, he only demonstrates surface understanding in this activity.

Laura's Venn diagram was similar to Kyle's, but she made a few more connections. For *mixed number* she wrote, "Mixed numbers have a whole number." For improper fraction she wrote, "The number at the top is bigger than the bottom." In the overlapping space she wrote, "They both are fractions," "You can make them in each other," and "They equle the same things." While her comments were not very clear, she made the connection that you can convert between the two types of numbers, and that you can have equivalent mixed numbers and improper fractions.

Megan's Venn diagram contained evidence of both surface knowledge and some deeper conceptual knowledge (see Figure 10). For *mixed numbers* she wrote, "Mixed numbers have 3 numbers Just like this 3 6/9 it has a whole number." For *improper fractions* she wrote, "Improper fractions have only 2 numbers and the top is Biger 7/6." In the overlapping space she wrote, "They are all fractions. They represent more than one whole." She demonstrated deeper knowledge when she wrote that they both represent values that are larger than one. While improper fractions can also equal one (e.g., 5/5), Megan appeared to have made deeper connections than did the other three focus students.

Yolanda's Venn diagram showed mostly surface and procedural knowledge. For *mixed numbers* she wrote, "You can make a mixed number into a improper fraction." For *improper fraction* she wrote, "You can make a improper fraction into a mixed number." And in the overlapping space she wrote, "There both fractions. The both equal the same things."

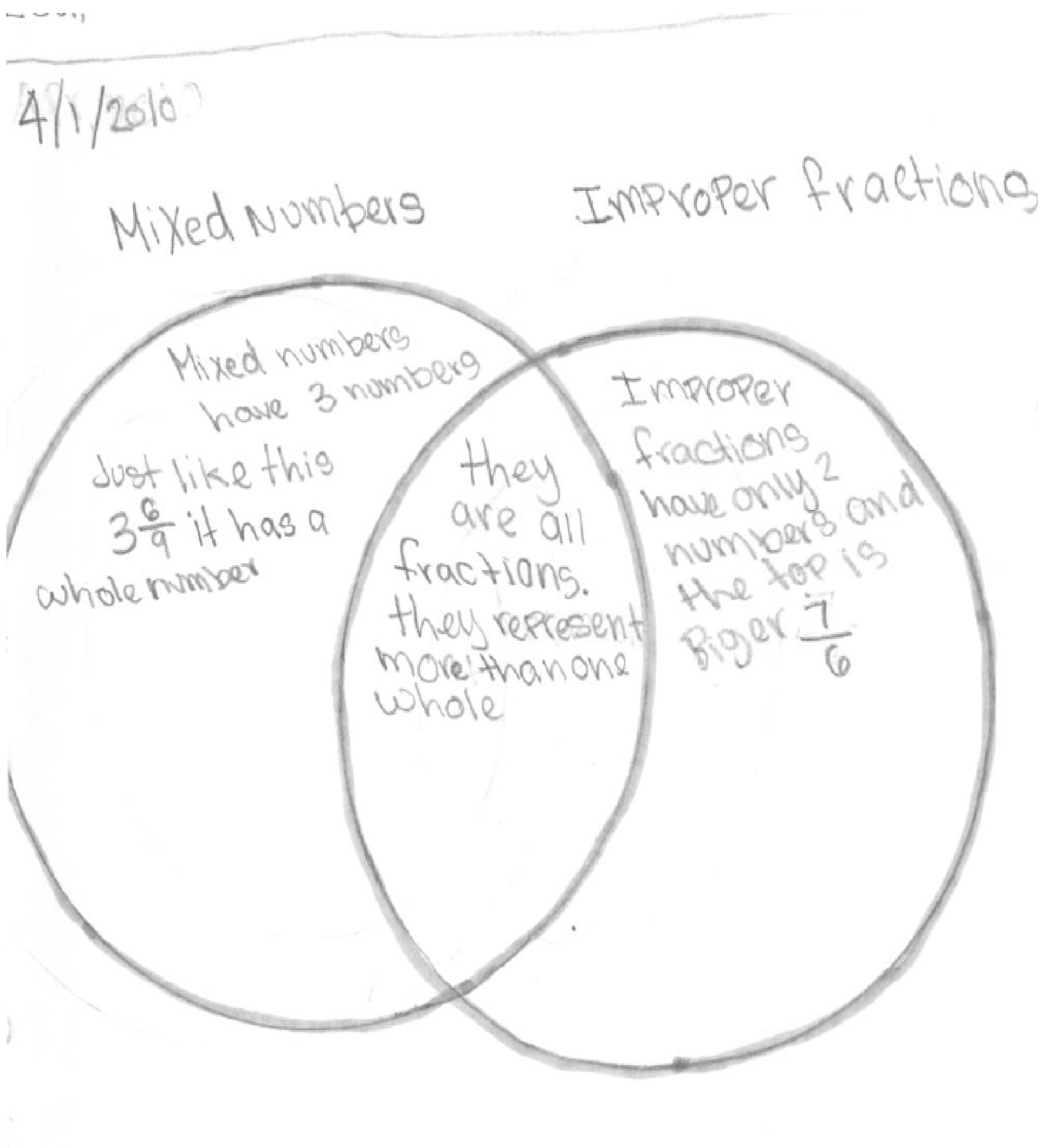

Figure 10. Megan's Venn diagram.

Content Knowledge

Although I looked at several aspects of mathematics in this study, helping my students gain deep mathematical content knowledge is at the forefront of my goals as a teacher. This theme comprises three categories: conceptual understanding, procedural knowledge, and mathematical accuracy. I felt strongly that I needed to know if my students were gaining the content knowledge included in this fractions unit.

One source that provided substantial data about whether my students were gaining content knowledge was the daily task papers. Students recorded their responses to the problems posed to them in each lesson on these papers. I created a rubric (see Appendix F) to help me score the solutions the students recorded. This rubric consists of scores ranging from 0–4, along with additional codes to further explain the students' work.

Kyle consistently received scores of 3s and 4s on his assignments. He received a few 2s, but overall he demonstrated mastery of the concepts taught in this unit. Laura's scores ranged from 0–4, with most of them being 3s. However, Laura had several additional codes next to her scores indicating her work samples often lacked sufficient work. Her task papers often had the correct answer written, but without enough evidence to show she understood. She appeared to understand based on her written answers and her spoken discourse when I asked her questions. I even wrote in my journal how I felt she was progressing. This is an excerpt from my journal written on March 24:

> Today we learned about adding mixed numbers. I was impressed as I saw kids explaining their answers in a much more detailed manner than they used to. Megan and Yolanda seem to need a lot of prodding. Laura is doing pretty well. Kyle does extremely well.

After comparing the focus students' collection of work and their post-performance assessments, it seems that Laura did not gain as much understanding as the other three students, even though she often appeared to understand more than Megan and Yolanda did during the unit.

Megan's scores also ranged from 0–4, with most of them being 3s. Megan's data showed a heavy need to be prompted or assisted as she solved problems. While she ultimately answered most of the problems correctly, she needed help for many of them. Yolanda's scores ranged from 0–3. She had the largest variability of scores, however, with more 1s and 2s than the other focus students had. She also had a variety of additional codes (a, b, and c) next to her scores, meaning she relied on assistance from me, and also occasionally relied heavily on her partners' assistance. Yolanda usually understood each concept to some extent, but lacked consistency.

Conceptual understanding. I want my students to have a deep conceptual understanding of the ideas taught in mathematics. In other words, I want my students to understand the mathematics at a conceptual level. Only Kyle and Megan demonstrated deep conceptual understanding through their oral discourse recorded in the 10 transcribed lessons (See Table 5). I saw evidence through their explanations that they understood more than just the procedure to solve a problem, but rather the reasons behind the steps. I looked at more than just clarity and completeness of responses to seek for conceptual understanding. I also noted whenever a student expressed new understanding of a concept. This usually occurred when a student responded with an exaggerated, "Ooohhhhh." All of the focus students experienced this kind of insight; however, this expressed new understanding was more common from Kyle and Megan, who both verbalized it five times. Laura verbalized new understanding only once, and Yolanda verbalized new understanding twice.

Table 5

Ways of Demonstrating Conceptual Understanding According to Focus Student

Ways of Showing Conceptual Understanding	Kyle	Laura	Megan	Yolanda
Demonstrates conceptual understanding through explanation	2	0	3	0
Expresses new understanding of a concept (e.g., ohhhhh)	5	1	5	2
Recognizes appropriate operation to use	5	5	8	2

Another situation that helped me look at student conceptual understanding occurred when the students recognized the appropriate operation to use when solving a problem (see Table 4). All four of the students were able to recognize when it was appropriate to add, subtract, multiply, or divide at least some of the time. During Lesson 8 Yolanda demonstrated this recognition as she explained how she solved the word problem, "Jonathan had rope that was 13 2/3 feet long. He cut off 4 1/5 feet. How long is the remainder of his rope?" Her recognition of the appropriate operation to use is evident in this exchange:

Me: Okay, can I ask you about this one? How did you find the seven fifteenths part?

Yolanda: Subtracted them. (March 26, 2010)

Through the context of the problem, Yolanda knew that subtraction was the appropriate operation to use in this situation.

There were occasions, however, when the focus students demonstrated a lack of prerequisite knowledge for this fractions unit. This lack of knowledge informed me of a gap in their conceptual knowledge. Kyle and Megan only had two such instances, while Yolanda had five, and Laura had nine. One of Yolanda's experiences in Lesson 7 clearly demonstrates a gap in conceptual knowledge. She stated, "It says how many pizzas do they eat in all. Is that subtracting?" (March 24, 2010). The word problem she was referring to stated, "Ben and Brenda had a pizza party. Ben ate 5 3/4 of a pizza, and Brenda ate 3 1/2 pizzas. How many pizzas did they eat total?" Yolanda was able to translate "How many pizzas did they eat total?" to "How many pizzas do they eat in all?" which is a different way of asking the same question. Yet, she lacked the prerequisite knowledge to immediately recognize this as an addition problem.

In Lesson 2, Laura showed her lack of prerequisite knowledge as she explored fractions using Cuisenaire rods. I tried to help Laura find a fraction equivalent to two thirds using the Cuisenaire rods. The following exchange shows our conversation:

Me: What if I have two of these green ones, and one of them is one third, right?

Laura: Mhmm, two ninths.

Me: K, but what size is this? This is a?

Laura: One third.

Me: Third, this is a third. So if I have two of them, that's—?

Laura: Two thirds.

Me: Two thirds, right? Can you use the white ones to make equal lengths here? K, so two thirds is equal to?

Laura: Two, two ninths.

Me: Are there only two of these?

Laura: Oh. Ummmm, nine <long pause> twoths? (March 4, 2010)

In this example, Laura demonstrated a lack of basic fraction knowledge. She needed extensive prompting to recognize that if one green rod was one third, and if we had two of them then we had two thirds. The second part of this conversation also demonstrates lack of fraction knowledge. Each white piece I was referring to was equal to one ninth, and we had six of them, so the correct response would have been “six ninths.” She originally said “two ninths,” and then, when prompted, changed her answer to “nine twoths.” She showed me in this exchange that she did not understand the numbers in fractions and what they mean. She did not understand what a numerator and denominator are. She also did not realize that “twoths” is incorrect usage, further demonstrating a lack of fraction knowledge.

Kyle and Megan appeared to have deeper conceptual understandings of fraction concepts than did Laura and Yolanda. Both Kyle and Megan had more recorded instances during the 10 lessons of demonstrating conceptual knowledge, and they both had fewer recorded instances of demonstrating a lack of fraction knowledge than did Laura and Yolanda.

Procedural knowledge. My goal is that my students have deep conceptual understanding of mathematics ideas, thus not relying solely on following a memorized procedure to work out mathematics tasks. However, procedures do help students solve problems efficiently, and having procedural knowledge in addition to conceptual knowledge is important. Rittle-Johnson et al. (2001) claimed that conceptual knowledge and procedural knowledge are interrelated. They stated, “Increases in one type of knowledge lead to gains in the other type of knowledge, which in turn lead to further increases in the first” (p. 347). Ideally, students will understand how to use efficient procedures to solve mathematics problems with a deep conceptual understanding of why the procedure works.

As I coded the lesson transcriptions, I noted each time a student demonstrated knowledge of a procedure. This demonstration of procedural knowledge often occurred when a student explained how she or he simplified a fraction or changed a mixed number to an improper fraction. I have record of all four focus students demonstrating procedural knowledge at some point during the 10 lessons. Kyle demonstrated this knowledge seven times, Laura six times, and Megan and Yolanda two times each. In Lesson 4 Laura demonstrated her procedural knowledge as she explained how she changed five and one third into an improper fraction: "'Cause you do five times three, fifteen, plus one." It is clear to me that she has learned and retained the procedure for changing mixed numbers to improper fractions. While Kyle and Laura may have appeared to have more procedural knowledge than did Megan and Yolanda, the number of occurrences became nearly equal when I added in the procedural knowledge occurrences from the pre- and post-interviews (see Table 6). It is clear to me that all four students had learned procedures to solve fraction problems, and they used them regularly.

Table 6

Number of Demonstrations of Procedural Knowledge According to Focus Student

Activity	**Kyle**	**Laura**	**Megan**	**Yolanda**
Lessons	7	6	2	2
Interviews	3	1	6	5
Total	10	7	8	7

Mathematical accuracy. Analyzing the students' justifications and procedural knowledge is very helpful to me as a teacher. However, at some point I must reflect on the mathematical accuracy of my students' work. Are they solving problems correctly? Are they relying on teacher prompts in order to find solutions? I have separated this Mathematical Accuracy category into two sections, with a total of five codes. First, I wanted to know if the students responded correctly or incorrectly. Whether they responded correctly or incorrectly, I examined the amount of prompting they received in order to come to that answer. If they answered incorrectly, I looked only at whether they came to that answer on their own or with prompting. If they answered correctly, I analyzed whether they needed extensive prompting, minimal prompting, or no prompting at all (see Table 7). For the purpose of this study, I examined answers to all questions a student orally answered, not just the final answer to a problem.

Table 7

Mathematical Accuracy with Different Levels of Prompting According to Focus Student

Accuracy and Prompting Level	Kyle	Laura	Megan	Yolanda
Correct with no prompting	7	8	3	5
Correct answer with minimal prompting	19	14	14	16
Correct answer with extensive prompting	3	12	25	10
Incorrect answer with no prompting	7	3	1	0
Incorrect answer with prompting	3	4	4	4
Total questions answered	39	41	47	35

Megan relied on teacher prompts the most, with 25 out of her 42 correct responses being reached with extensive prompting, and only three reached with no prompting. Kyle, on the other hand, only needed extensive prompting for three of his correct responses. Laura's and Yolanda's reliance on teacher prompting appeared approximately equal.

The four students answered correctly many more times than they answered incorrectly. Kyle relied on extensive prompting the least, but he also incorrectly answered more questions than the other three students. He answered three questions incorrectly with prompting, and the three girls each answered four questions incorrectly with prompting. Kyle answered seven questions incorrectly without prompting, while Laura answered three and Megan answered one. It appears that all of the focus students can answer questions correctly with support.

Mathematical Confidence

Even before selecting my focus students to participate in this study, I was aware that my pool of potential participants had varying levels of mathematical competence. I also suspected that along with different levels of competence would come different levels of confidence. Although I did not have fifth grade mathematics CRT scores for Kyle, and therefore did not have a formal measure that helped me compare him to my other focus students on fifth grade mathematics understanding, he had demonstrated a high level of mathematical competence throughout the first several months of the school year. Because of Kyle's demonstrated high performance, I expected him to have higher confidence levels than the other three focus students, who had performed at much lower levels than Kyle did all year. My predictions proved partially true. I recorded the number of times the students expressed confidence in their mathematical understanding, and Kyle expressed his confidence seven times. Yolanda expressed confidence three times, while Laura and Megan expressed confidence only once each during the 10-lesson period.

Yolanda's expressions of confidence came near the end of the unit. Her confidence seemed to grow as she gained competence in finding common denominators using a strategy one of her group members taught her. This strategy simply involved listing the multiples of the two denominators until a common multiple was found, but it seemed to influence Yolanda's confidence greatly. My perception of Yolanda's strengthening confidence, as well as those of my other focus students, can be seen in my journal entry from March 26:

> I was very pleased that my focus students were eager to participate. Yolanda even came to the board to share a problem. I think that was a boost for her self-esteem. Kyle also shared a problem on the board, and Megan shared an answer from her seat. The kids seem confident with the concepts.

The students showed their confidence by their willingness to share their solutions in class.

Conversely, I also recorded some events that signified to me a lack of confidence in mathematical understanding. This lack of confidence was presented in one of three ways. First, if a student appeared unsure in response, e.g., if he or she answered in question form, I interpreted that as a lack of confidence in their mathematical understanding. Next, if a student responded with hesitance, which typically came in the form of "ummm" or "uhhh," I construed that as another way of demonstrating lack of confidence. The final demonstration of lack of confidence presented itself when the students verbally claimed they did not know or understand.

Megan appeared unsure of her response by answering in question form 17 times. Laura and Yolanda both appeared unsure 14 times, while Kyle only appeared unsure nine times. However, Kyle responded with hesitance the most, with 41 responses including "ummm" or "uhhh." Laura responded with hesitance thirty-four times, Megan twenty-three times, and Yolanda only ten times. Additionally, Kyle stated that he did not know or understand the most often, with 13 such responses. Megan made similar statements 10 times, while Laura and Yolanda only made such statements twice each. In this study, the students who appeared to have deeper understandings of the concepts are the ones who claimed not to understand the most often.

Use of Metacognition

Another theme that emerged from the transcriptions was student use of metacognition. There were two types of situations that surfaced as I reviewed the transcriptions that fit into this category. First, if a student recognized a gap in her or his understanding, I recorded this use of metacognition. The second use of metacognition occurred when a student recognized an error and then self-corrected. Yolanda and Laura each recognized an error and then self-corrected once, while Kyle recognized an error and self-corrected three times and Megan self-corrected four times. Yolanda and Megan both recognized gaps in their understanding twice, and Kyle recognized a gap once.

Mathematics Vocabulary

Mathematics vocabulary is central to my study. I was deliberate in teaching mathematics vocabulary using research-based strategies. It pleased me to hear my students using the vocabulary words correctly in their conversations. Kyle used mathematics vocabulary correctly far more frequently than did the three other focus students (see Table 8). Yolanda expressed confusion about vocabulary words most frequently, followed by Laura and then Megan. This order is nearly the reverse of the frequency of correct vocabulary usage. Confusion was expressed when a student asked about a vocabulary word or if she or he used a vocabulary word incorrectly.

Table 8

Instances of Vocabulary Usage According to Focus Student

Vocabulary Usage	Kyle	Laura	Megan	Yolanda
Correct vocabulary usage	44	24	20	17
Expressed confusion about vocabulary	4	7	5	9

I also examined how often students used imprecise words instead of mathematics vocabulary words in conversation (e.g., saying "the top ones" when referring to numerators). Kyle demonstrated use of imprecise words in Lesson 2 when he said, "Uh, the bottom ones, uh, they're like, you multiply them by two, like top one by two, then you get two sixths" (March 4, 2010). Use of imprecise words did not occur very often during the 10 lessons. Kyle used imprecise words twice, Laura and Megan each used imprecise words once, and Yolanda never used imprecise words. However, all four students used imprecise words in the post-interview as they explained their solution to the problem they were given.

Using an online vocabulary profiler (Cobb, 1994), I was able to take a closer look at the kinds of vocabulary my focus students used. This website categorized each word spoken by the students into four categories: first 1000 most common words, second 1000 most common words, academic words, and words not on any of those three lists. Many of the words that were not on a list were domain-specific mathematics words. I examined the words spoken by my four focus students that were on the academic word list and the mathematics words that were not on any list. I listed all of the mathematics-related words and counted the number of times each word was used (see Table 9). Once again, *tokens* refer to the total number of words spoken, even words that are repeated. *Types,* on the other hand, refer to the number of unique words spoken. I found many similarities in the kinds of vocabulary used by the students. For example, all four focus students used the words *equivalent* and *plus*, which were both on the academic word list. All four students used the words *fraction* and *denominator* many times, although Megan was the only student to use the word *numerator*. Many of the other mathematics words that were not on any list were fraction names (e.g., fifths, ninths) and names of polygons (e.g., trapezoid, rhombus). The students used the names of polygons in Lesson 1 as they explored fraction concepts with pattern blocks.

Table 9

Frequency of Mathematics Vocabulary Tokens and Types According to Focus Student

	Kyle		**Laura**		**Megan**		**Yolanda**	
Vocab	Academic List	Math-Specific	Academic List	Math-Specific	Academic List	Math-Specific	Academic List	Math-Specific
Tokens	6	54	5	49	6	44	3	29
Types	4	13	2	11	2	11	2	12

Of the total words spoken by each student, I computed the percentage of words that were mathematics vocabulary. For this percentage, I also included mathematics words that occurred in the second 1000 most common words. These included words such as *multiple*, *pair*, and *compare*. Kyle had the smallest percentage, with 4.1% of his total words being mathematics vocabulary. Megan had 5.7% of her words being mathematics vocabulary. Laura was next, with 6.5% of her total words being mathematics vocabulary. Yolanda had the highest percentage of words being mathematics vocabulary, with 7.1%. These percentages do not take into account whether the usage was correct, only that the vocabulary words were spoken.

All four focus students used the vocabulary that was explicitly taught during this lesson. Their spoken and written discourse was clear and accurate as precise mathematics vocabulary words were used. Some imprecise vocabulary was also used, along with expression of confusion about mathematics vocabulary.

These results give insight into the mathematical thinking of the four English learners who participated as focus students in this study. The progress and growth in their mathematical thinking can be seen in the data that were collected. Conclusions and implications of the results are discussed in the next chapter.

Chapter 5

Conclusions and Recommendations

In conducting this study, I examined what happened to the mathematical thinking of English learners as they were taught mathematics vocabulary through research-based strategies. I have been unable to locate similar studies in the literature; therefore, I do not have other studies with which to compare my results. Because I did not gather data on the mathematical thinking of my native-English speaking students, comparisons cannot be made between native-English speakers and my focus students, who were all English learners. This study was meant to describe what happened in this particular situation. Perhaps with additional studies similar to mine, the results of this study could one day be a part of a body of research providing a baseline for describing the nature of the mathematical thinking of English learners.

Answering the Research Question

Kyle, Laura, Megan, and Yolanda all gained procedural and conceptual knowledge throughout this unit on fractions. Their pre- and post-performance assessments give evidence of this growth. Their performance at the end of the unit was at levels similar to the rest of the class. Based on the scores of my class, both my native-English speakers and my English learners successfully learned the mathematical content involved in this fractions unit. It appears that my focus students did not fall behind.

All four students used the mathematics vocabulary words that were taught explicitly in class. They engaged in discourse, in varying levels, about mathematics, and used both mathematics vocabulary and imprecise, everyday language to describe their ideas. While the mathematics tokens varied according to the individual student, the types of mathematics vocabulary used were similar among the four students. There were only a few words that were used by one student and not the others. The word wall appears to have contributed to the focus students' consistent use of mathematics vocabulary. I saw Megan look up at the word wall in search of a word on multiple occasions. I also transcribed instances when focus students referred to the word wall in spoken discourse.

The students created pictorial representations fairly consistently across lessons. I strongly encouraged the students to draw pictures to help them solve problems. At first some of the focus students appeared uncomfortable drawing pictorial representations, but over the course of the unit, drawing pictorial representations came much more naturally to the students. The students appeared to gain conceptual understanding when they drew pictorial representations.

All four focus students attempted to give clear justifications for answers. While they all were able to give some clear, complete justifications, I more commonly transcribed justifications lacking clarity and accuracy. The students struggled to explain why answers were correct. This difficulty in explaining clearly could be attributed to many potential factors. First, if students lack conceptual understanding, they understandably would have difficulty explaining those concepts. Additionally, I think these students may not have had much practice in justifying their answers. Many mathematics classes are taught through direct instruction, which typically does not allow students much opportunity to interact with others. Also, individuals may have more difficulty making sound arguments if they do not have substantial command of the language. Because the four focus students scored in the intermediate range on the UALPA, we can conclude that they had some difficulty with the academic language used in giving clear justifications for answers.

The students put their ideas in writing as they completed journal entries about fractions. In most instances, the students responded to structured writing prompts with well-constructed sentences. These written responses differed from their spoken discourse, which often consisted of incomplete thoughts. Having the opportunity to write their thoughts seemed to give them a chance to develop their ideas more carefully. When the students were asked to write their think-pair-share responses first, more students participated in the "share" part of the activity. While these written responses still sometimes lacked evidence of understanding, there usually was an added level of clarity in their expression.

I think one of the greatest successes of this study is the students' expression of enjoyment during this unit. While this information is not directly related to my study, it adds interesting insight to the students' perspectives. At the end of this fractions unit, I asked my students to raise their hands if they felt they understood fractions better then than they had a month before. Every hand in my class was raised. Not only did they feel they learned a considerable amount, they also expressed excitement and joy when referring to learning about fractions. At the end of the school year, a few months after I taught this unit on fractions, I asked my students to reflect on what they learned in mathematics that year. I asked them to think of things they particularly liked learning. One student immediately said she liked what we did with fractions, and several other students voiced agreement. The students noticed a difference in the way I taught fractions, and they liked it. This response from my students gives me a strong desire to improve my implementation of inquiry teaching so that I may continue to provide other students with opportunities for the same excitement about mathematics that these students experienced. I already have substantially improved my planning and carrying out inquiry through the implementation of this study.

While there were many positive outcomes during this unit of study, there were also a few less desirable outcomes. For example, I observed that these sixth graders were heavily reliant on procedural knowledge, in spite of my efforts to help the students develop conceptual knowledge. This reliance on procedures can be attributed to many factors. First, many of my students came to sixth grade already knowing some procedures relating to fraction problems. When presented with a task, some students jumped immediately to using a procedure they remembered learning the previous year. These students sometimes influenced the students near them also to use the procedure. This situation occurred often with Yolanda and her partner. One of Yolanda's partners was both competent and confident in her mathematics abilities. She was recorded explaining procedures to Yolanda many times. Perhaps this learning of the procedures limited Yolanda's desire to develop conceptual understanding.

While some students had learned procedures previously, I also worry that I may have jumped to teaching or explaining procedures too quickly after letting the students explore. I am new at teaching inquiry for a whole unit, which may have influenced my ability to teach inquiry well. I had a strong desire to help my students gain conceptual knowledge. When students presented their solutions to the class, many students explained using procedures they remembered from previous years. In an attempt to help my students understand why the procedure works, I would explain the procedure conceptually, often drawing pictures. My attempt to explain the procedures conceptually may have had negative effects, however, if students only focused on the procedure. I now recognize that students should be more responsible for explaining the procedures to me, and not the other way around.

Some of my focus students demonstrated a real lack of confidence in their mathematics abilities. Yolanda was recorded telling her partner twice, "It may be easy for you, but it's not for me." In spite of this perception that the mathematics was difficult for her, Yolanda exhibited a true desire to learn and understand the material. Her confidence grew over the course of this unit, as shown when she eventually volunteered to solve a problem on the board. She gained self-assurance as she was able to apply a strategy that her partner taught her to use to find least common denominators correctly. She listed the multiples of each denominator until she found the least common multiple. This simple strategy helped her gain both confidence and competence in mathematics.

Yolanda exhibited very limited mathematics knowledge of fractions in both her procedural pretest and pre-interview. She spoke much less frequently than did the other three focus students. She rarely participated in group discussions or think-pair-shares. Yet she was able to make considerable growth in understanding during this fractions unit. Much of her growth came near the end of the unit and may have been tied to her growing confidence and ability to find common denominators. Her vocabulary usage also seemed to develop substantially as the unit neared its end.

Laura, on the other hand, appeared to be making progress during the unit through her written responses, and even her conversations with her partner; however, evidence shows that Laura was relying heavily on the understanding of her partner and did not fully understand everything that she appeared to comprehend. Laura did not draw attention to herself and rarely asked for help. She appeared to prefer going unnoticed and not learn the concepts than to draw attention to her lack of understanding. Her post-performance assessment demonstrates less growth and accuracy than that of the other three focus students. Her performance on the procedural part of the assessment was very inconsistent. She correctly followed procedures such as simplifying or finding common denominators for some items, but she did not follow those procedures for all items in which they would apply. Based on Laura's fifth grade Mathematics CRT score of 1 (indicating minimal understanding), we can assume that Laura had been largely unsuccessful in many previous mathematics experiences. Her poor previous performance may indicate prerequisite mathematical concepts that had not yet been mastered. Laura appears to be capable of learning mathematics procedures but struggles with knowing when to apply them.

While this information is not directly related to this study, I observed that Megan worked hard in all subjects and had a strong desire to succeed, even though academics seemed to be difficult for her. She often claimed that she did not understand or a problem was difficult, but she always attempted to work through her difficulties. I think her perseverance was evident throughout this unit. She began the unit with very little fraction knowledge but had made tremendous growth by the end. She demonstrated conceptual knowledge several times and made connections between concepts and procedures.

Although Yolanda and Megan both made substantial improvements in knowledge, I fear too much of their knowledge was procedural, without enough conceptual understanding to support this procedural knowledge. This lack of conceptual knowledge was evident in their attempts to solve the post-interview word problem, which involved subtracting mixed numbers with regrouping. Both girls showed that they remembered a procedure to some extent; however, their work demonstrated a lack in conceptual understanding. This situation shines light on a major dilemma of memorizing procedures: If a person forgets even one step to a procedure, he or she likely will solve the problem incorrectly. Conversely, if that person has deep conceptual understanding, she or he is more likely to discover a conceptual way to find a solution because conceptual knowledge is generalizable and not tied to a specific type of problem (Rittle-Johnson et al., 2001).

Kyle demonstrated substantial knowledge, both procedural and conceptual, throughout this unit. The number of words he used to talk about mathematics was much higher than the other three focus students. Kyle had often expressed his enjoyment of mathematics, and he had certainly performed at high levels mathematically. Kyle talked about mathematics using more tokens than the others, perhaps because he knew what needed to be said. He was more familiar with mathematics concepts, and, therefore, was able to make more personal connections to the mathematics. With increased mathematics understanding may come increased mathematical discourse. When the other focus students develop stronger understanding of mathematics concepts, perhaps their discourse will also strengthen and increase.

Reflection on Implementation

This unit on fractions was my first experience using many of the research-based vocabulary strategies that were employed. I learned a great deal about these strategies through their implementation. While there are many aspects of the strategies that worked well, there are also several changes I would make in how I implement these strategies in my classroom. The word wall appeared to be a great success. Students referred to the word wall often throughout their discussions. I typically reminded the students about the words on the word wall at the beginning of each lesson, even if we did not add new words that day. This reminder may have kept the word wall fresh in the students' minds. The word wall was a consistent part of each lesson, and it appears the students felt comfortable using the vocabulary words.

The picture dictionaries could have been a very useful strategy had I implemented the strategy more carefully. When I first asked the students to draw pictures explaining each word from the word wall, I heard several complaints and remarks of confusion. They did not want to draw pictures. Instead of taking these comments and using them to guide my instruction, I simply repeated my encouragement to the students that they would be able to accomplish what I asked of them. I see now that many students decided to created entries similar to what they see in traditional dictionaries. I think if I had shown some examples of pictures defining mathematics vocabulary, the students would have had a better idea of what was expected. Also, if I had given them additional time to update or improve their entries throughout the unit, I likely would have seen entries of better quality. Although the use of picture dictionaries was not very successful, the students did demonstrate understanding through the pictures they drew as they solved the tasks for each lesson.

My initial reaction regarding the outcome of the think-pair-shares was that they were not very successful. I observed multiple occasions where students failed to share their thoughts, which led me to wonder whether or not those students even thought about the question. However, after analyzing the responses during the think-pair-shares, I now conclude that the think-pair-shares can really draw out excellent ideas and discussion if the students participate. When the students felt comfortable sharing their thoughts with their partners, they had the chance to practice using the mathematics vocabulary in a safe environment. If the students had more practice with this kind of activity, they likely would eventually feel more comfortable participating.

I had great plans to have my students create graphic organizers to help them make connections between mathematical concepts throughout the unit. However, I failed to plan carefully enough to incorporate the many graphic organizers I had hoped to use. I was able to include only one graphic organizer, a Venn diagram, in this unit. While I was disappointed with the quantity of graphic organizers I used, I was moderately pleased with the quality of these Venn diagrams. Many students made only surface connections; however, some students were able to make deeper connections. With practice and repeated use, I likely will be better at implementing graphic organizers, and also, my students will be better at knowing how to complete them. Graphic organizers, when appropriately used, have the potential to become a helpful tool for students to use in learning mathematics vocabulary (Monroe & Orme, 2002).

Most of the journal entries my focus students wrote were in response to prompts I had given them. I would like to help my students write about mathematics without being prompted to, or at least without being prompted about what to write. Perhaps with continued use of journal writing, students will become less reliant on prompts. Although I had hoped my students would have written more journal entries, I was pleased with what they wrote.

This study has had immense influence on my practice. After seeing how comfortable the students became with fractions, and how much they enjoyed learning through inquiry methods, I have a renewed desire to improve my teaching of mathematics through inquiry. I also am much more aware of the vocabulary my students use in their discourse, and I regularly seek ways to help my students improve their vocabulary.

Implications

The results and conclusions presented in this study are specific to the four focus students and the context of this study. In order to gain a better understanding of English learners and their mathematics vocabulary development, additional studies are needed. One possibility for a future study involves examining how the mathematical thinking of English learners compares to that of native-English speakers in a context similar to this study. Another possibility for future research is to compare a class of students learning mathematics using research-based vocabulary strategies to a class learning mathematics without these strategies.

These findings also carry implications for teaching. The students in my class expressed increased confidence in their mathematics abilities and increased excitement about mathematics after having participated in this inquiry unit on fractions. Three of the four focus students also showed substantial growth and understanding during this unit as measured by the pre- and post-performance assessments. These results offer excellent reasons to teach using inquiry methods and to be explicit in vocabulary instruction. This study was not an experiment, and therefore I cannot make causal claims; however, it may be that in this situation, the explicit focus on mathematics vocabulary during this unit not only encouraged students to speak using precise vocabulary, but also helped strengthen their understanding of the mathematics content.

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Appendix A

Consent to be a Research Subject

Introduction

This research study is being conducted by Hilary Hart as a part of her master's thesis to examine the mathematical thinking of 6th grade English learners as they are taught mathematics vocabulary through research-based practices. Your child was selected to participate because he/she is enrolled in Hilary Hart's 6th grade mathematics class and is either an English learner or will be working in a small group with an English learner. The research study will be supervised by Dr. Eula Monroe, Professor in Teacher Education in the David O. McKay School of Education at Brigham Young University.

Procedures

When consent is given, your child will participate in this research study as a part of mathematics class during the 2009-2010 school year. Each participant will contribute to class and group discussions during a mathematics unit on fractions. In addition, 3-5 English learners will be interviewed regarding their use of mathematical language. Participants may have their voices audio-recorded. Audio recordings will be transcribed. Student work samples will be photocopied and returned to the participant.

Risks/Discomforts

There are minimal risks for participating in this study. However, your child may feel uncomfortable being audio-recorded.

Benefits

It is not anticipated that your child will receive any direct benefits from participating in the study. However, your child will be participating in a learning environment in which the language of mathematics will be emphasized.

Confidentiality

All information provided will remain confidential and will be reported with no identifying information. When data are reported for individual participants, pseudonyms will be used. All data, including student work samples, audio-recordings, and transcriptions will be kept in a locked cabinet or on a password-protected computer. Only those directly involved in the research will have access to them.

Compensation

Your child will not be compensated for his/her participation in this study.

Participation

Students who choose not to participate in the study will participate in 6th grade mathematics as a regular part of instruction. However, their work will not be copied, audio tapes of their discussions will be erased and not used for the study, and observational notes of the child will be excluded from the study. You have the right to excuse or withdraw your child from participating in the study at anytime without jeopardy to your child's class status or grade.

Questions about the Research

If you have questions regarding this study, you may contact Hilary Hart <personal information omitted> or Eula Monroe <personal information omitted>

Questions about your Child's Rights as a Research Participant

If you have questions you do not feel comfortable asking the researcher, you may contact the Brigham Young University IRB Administrator, <personal information omitted> <personal information omitted>

I have read, understood, and received a copy of the above consent and desire of my own free will that my child participate in this study.

[ ] Yes, I would like my child to participate in this study.

[ ] No, I would not like my child to participate in this study.

Signature: ______________________________ Date: ____________________

MSE Approved
301 MCKB
Jan 06, 2011

Assent for a Research Subject

Dear Student,

Miss Hart is doing a research study as a part of her master's degree. She is going to be doing a few extra things during math, such as audio-recording classes. If you choose to participate, your voice may be recorded and what you say may be used in her study. You may also be asked to participate in interviews with Miss Hart. This may cause you to feel uncomfortable. You can choose whether you want to be a part of this study or not. If you choose to participate, you may later decide to withdraw. This will not affect your grade.

[ ] Yes, I would like to participate in this study.

[ ] No, I would not like to participate in this study.

Student Signature: ______________________________ Date: ____________________

MSE Approved
301 MCKB
Jan 04, 2011

Consentimiento para ser Sujeto a Investigación

Introducción
Este estudio de investigación se esta llevando a cabo por Hilary Hart como parte de su tesis de maestría para examinar el pensamiento matemático de estudiantes de ingles de 6to grado mientras aprenden vocabulario matemático mediante un programa educativo basado en investigación. Su hijo/a fue seleccionado/a porque es alumno/a en el salón de 6to grado de matemáticas de Hilary Hart y esta aprendiendo ingles como segundo idioma o estará trabajando en un grupo pequeño con un estudiante de ingles como segundo idioma. La investigación también será supervisada por Dr. Eula Monroe, Profesora de Enseñanza de Educación en la Escuela de Educación David O. McKay de la Universidad de Brigham Young.

Procedimiento
Cuando se da consentimiento, su hijo/a participara en este estudio de investigación como parte de la clase de matemáticas durante el ano escolar 2009-2010. Cada participante contribuirá a la clase y la platica del grupo durante una unidad de lecciones sobre quebradas. Además, 3-5 estudiantes de ingles como segundo idioma serán entrevistados acerca de su uso del lenguaje matemático. A los participantes se les puede grabar sus voces. Grabaciones de audio serán transcritos. Ejemplos de trabajo de los alumnos serán copiados y devueltos a los participantes.

Riesgos/Incomodidades
Los riesgos son mínimos por participar en este estudio. Pero, su hijo/a podría sentirse incomodo/a al ser grabado por audio.

Beneficios
No se anticipa que su hijo/a se reciba ningún beneficio directo por participar en este estudio. Pero, su hijo/a participara en un ambiente en que el lenguaje matemático será enfatizado.

Confidencialidad
Toda la información se mantendrá confidencial y se reportara sin ningún dato personal. Cuando se reportan datos por participantes individuales, se usaran seudo nombres. Toda la información, incluyendo ejemplos de trabajo estudiantil, grabaciones de audio, y transcritos se guardaran en un gabinete con candado o en una computadora protegida con contraseña. Únicamente los que están involucrados directamente tendrán acceso a ellos.

Compensación
Su hijo/a no será compensado/a por su participación en este estudio.

Participación
Estudiantes que eligen no participar en el estudio participaran en matemáticas de 6to grade como parte de su instrucción normal. Pero, sus trabajos no serán copiados, grabaciones de sus pláticas serán borradas y no usadas en el estudio, y notas de

observación serán excluidas del estudio. Usted tiene el derecho se excluir o sacar a su hijo/a del estudio en cualquier momento sin afectar la posición o calificación de su hijo/a en la clase.

Preguntas sobre la investigación
Si tiene alguna pregunta sobre esta investigación, puede contactar a Hilary Hart <personal information omitted> a Eula Monroe <personal information omitted>

Preguntas sobre los derechos de su hijo/a como participante de investigación
Si tiene alguna pregunta y no se siente cómodo al preguntarle a la maestra, puede contactar al administrador del consejo de investigación de la Universidad de Brigham Young <personal information omitted>

Yo he leído, entendido, y recibido una copia del consentimiento arriba y deseo de mi propia y libre voluntad que mi hijo/a participe en este estudio.

[ ] Sí, me gustaría que mi hijo/a participara en este estudio.

[ ] No, no me gustaría que me hijo/a participara en este estudio.

Firma: ______________________________ Fecha: __________________

MSE Approved
301 MCKB
Jan 04, 2011

Asentimiento para un Sujeto de Investigación

Querido Estudiante,

Miss Hart se va a llevar a cabo una investigación como parte de su maestría. Ella va a hacer algunas cosas extras durante la clase de matemáticas. Si eliges participar, se puede grabar tu voz y lo que dices se puede usar en su estudio. También te puede pedir que participes en algunas entrevistas con Miss Hart. Esto te puede causar incomodidad. Tú puedes escoger si quieres ser parte de este estudio o no. Si eliges participar, te puedes salir después. Esto no afectara tus calificaciones.

[ ] Sí, me gustaria participar en este estudio.

[ ] No, no me gustaria participar en este estudio.

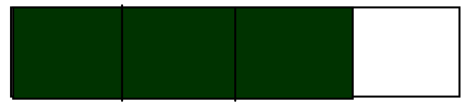
Firma del estudiante: ______________________________ Fecha:____________________

Appendix B

Fractions Procedural Pre-assessment

Name: ________________________________

1. Write two fractions that are equivalent to ½: ___________, ____________

2. Draw two fractions that are equivalent to: 

3. Write 4/12 in simplest form: ___________

4. Prove that 6/9 is equal to 2/3:

Add:
5. 1/7 + 3/7 = ____________

6. 2/5 + 1/10 = ___________

7. 3/4 + 1/6 = ____________

8. 2/3 + 3/5 = ____________

9. 1¼ + 2½ = ____________

Subtract:
10. 6/8 – 3/8 = ___________

11. 7/8 – 3/4 = ___________

12. 3/4 – 3/9 = ___________

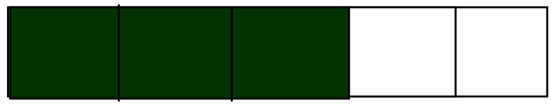
13. 3¾ - 2¼ = ____________

14. 5¼ - 3½ = ____________

Fractions Procedural Post-assessment

Name: _________________________________

2. Write two fractions that are equivalent to ¾ : ___________, ____________

2. Draw two fractions that are equivalent to: 

3. Write 4/10 in simplest form: ___________

4. Show that 6/8 is equal to 3/4:

Simplify your answers to the following questions:
Add:
5. 1/5 + 3/5 = ____________

6. 1/4 + 3/8 = ___________

7. 1/4 + 2/6 = ____________

8. 1/3 + 3/7 = ____________

9. 3 1/8 + 2½ = ____________

Subtract:
10. 6/7 – 4/7 = ___________

11. 5/6 – 2/3 = ___________

12. 1/2 – 2/7 = ___________

13. 5¾ - 2 1/8 = ____________

14. 4 1/6 - 1½ = ____________

Appendix C

Interview Protocol

Preinterview Questions for Focus Students:

Kyle: My dog is 5 ½ years old. My cat is 3 ¾ years younger than my dog. How old is my cat?

Laura and Megan: My recipe calls for ½ cup white flour and ¼ cup whole-wheat flour. How much flour do I need in total for my recipe?

Megan and Yolanda: My recipe calls for $^{1}/_{8}$ cup white flour and $^{5}/_{8}$ cup whole-wheat flour. How much flour do I need in total for my recipe?

Yolanda: I made a pan of brownies and my brother ate $^{4}/_{12}$ of them. Show me a different way to write the fraction of brownies he ate. (You may draw pictures)

Postinterview Questions for Focus Students:

Kyle, Megan, and Yolanda: My dog is 7 1/3 years old. My cat is 4 7/9 years younger than my dog. How old is my cat?

Megan and Yolanda: My dog is 5 5/6 years old. My cat is 3 1/3 years younger than my dog. How old is my cat?

Laura: My dog is 4/5 years old. My cat is 1/4 years younger than my dog. How old is my cat?

Appendix D

Lesson Plans

Fractions Lesson 1

Topic: Relationships Between Fractions	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/2/10**

Goals: Content Goal: Students will explore relationships between fractions using pattern blocks and will demonstrate the meanings of fractions as parts of a whole. They will explain the relationships that they discover in their logs. Vocabulary Goal: Students will use the words "fraction," "equivalent," "numerator," and "denominator" in conversation and in their logs.	**Materials Needed:** Promethean Board, pattern blocks, student logs, pencils, activity sheet, computers
State Core Connection: Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.	

Description of the Mathematical Task: Students will use pattern blocks to explore the relationships among different shapes and sizes.

<u>**Launch**</u>	<u>**Explore**</u>	<u>**Summarize**</u>
Approx Time: 2 minutes Cycle 1: Prior to the launch I will place the four vocabulary words on the word wall. I will pronounce each word for the class and will tell them we will be using these words throughout the lesson. I will give	**Approx. Time: 5-10 minutes** Cycle 1: I will monitor students as they use the pattern blocks to explore relationships. I will ask the following questions: 1. Is there a way to represent the red trapezoid using blue and green pattern	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will begin the "summarize" segment with a think-pair-share. The question will be "What relationships did you notice between the different pattern blocks?"

each group a set of pattern blocks. I will tell them I want them to explore the relationships among the green triangles, the blue rhombuses, the red trapezoids and the yellow hexagons. I will tell them to record their thoughts and any relationships they see in their logs.	blocks? 2. Can you cover the red trapezoid using only one color? What does this tell us about the relationship between the blue rhombus and the green triangle? 3. Are there other ways to represent various pattern blocks using more than one color pattern block? *As the students continue to explore the pattern blocks in their groups, I will be selecting students to share their findings with the class. Students will be selected if they have discovered something unique, or if their findings help bring out the big mathematical ideas.	On the Promethean Board, students will draw and explain the relationships they noticed between the pattern blocks. I will make sure the student presentations lead the class to the big mathematical ideas. If students do not make this connection, I will point out that "equivalent" shapes can be made by combining smaller shapes. For example, two green triangles make one blue rhombus, and three blue rhombuses make one yellow hexagon. I will then give the students 3-5 minutes to draw representations in their logs.
<u>**Launch**</u> **Approx Time: 2 minutes** Cycle 2: I will tell the students that they are going to continue to explore the relationships between pattern blocks, but they will now need to use the words "fraction," "equivalent," "numerator," and "denominator" as they explore and discuss the relationships. I will give them each a paper with questions that	<u>**Explore**</u> **Approx. Time: 10-15 minutes** Cycle 2: I will monitor students as they continue to use the pattern blocks to explore relationships. I will guide them to use the words "fraction," "equivalent," "numerator," and "denominator" in their conversations and explanations. I will ask the following questions as I monitor each group:	<u>**Summarize**</u> **Approx. Time: 10-15 minutes** Cycle 2: This time we will connect the fraction names to the shapes. For example, if the yellow hexagon is our whole, the red trapezoid is 1/2, the blue rhombus is 1/3, and the green triangle is 1/6. However, if the red trapezoid is one whole, then the green triangle is 1/3. I will ask the students questions like "If

help guide their explorations.	1. How many green triangles are in one blue rhombus? The green triangle is what fraction of the blue rhombus? What part of the fraction is the numerator? What does the numerator in this fraction mean or represent? What part of the fraction is the denominator? What does the denominator in this fraction mean? 2. How many green triangles are in one red trapezoid? The green triangle is what fraction of the red trapezoid? What part of the fraction is the numerator? What does the numerator in this fraction mean or represent? What part of this fraction is the denominator? What does the denominator in this fraction mean? *I will once again be looking for students who have strong ideas to share with the class during the summarize portion of the lesson.	the red trapezoid is one whole, how much is the blue rhombus? The yellow hexagon? I will then give the students time to write a summary and draw representations of what they learned about fraction relationships in their logs.
Accommodations for Diverse Learners: English as a Second Language: I will write the words "fraction," "equivalent," "numerator," and "denominator" on the word wall so the EL students can become familiar with the form of the	English as a Second Language: I will take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	English as a Second Language: I will make sure that through the discussion the correct definitions of the focus words are stated clearly.

words.		
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Are there enough pattern blocks for each group? As a way to assess each child individually, I will give an additional task that they must solve by themselves. I will remind the students to use what they have learned through their exploration.	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *Fractions can be described as parts of a whole. *You can group smaller pattern blocks together to make them "equivalent" to larger blocks. *The denominator of a fraction shows us how many pieces the whole is split into. *The numerator of a fraction tells us how many of those pieces we are talking about.

Lesson 1 Activity

Name: _________________________ Date: ________________________

Shape Relationships

1. What relationships can you find between the triangles and the rhombus?

2. What relationships can you find between the triangles and the trapezoid?

3. What relationships can you find between the triangles and the hexagon?

4. What relationships can you find between the rhombuses and the hexagon?

5. What relationships can you find between the trapezoids and the hexagon?

Lesson 1 Assessment

Name: ____________________________ Date: _________________________

Shape Exploration Assessment

What is the relationship between the trapezoid and the rhombus?

____

____

____

Use the other blocks to explain your answer:

____

____

____

____

____

____

Draw a picture that shows your answer:

Fractions Lesson 2

Topic: Fraction Equivalence	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/4/10**
Goals: Content Goal: Students will explore fraction equivalence. They will be able to explain why two fractions are equivalent. Vocabulary Goal: Students will use the words "fraction," "equivalent," "numerator," and "denominator" in conversation and in their logs. **State Core Connection:** Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.		**Materials Needed:** Promethean Board, Cuisenaire Rods, student logs, pencils, activity sheet, computers	

Description of the Mathematical Task: Students will use Cuisenaire Rods

Launch	**Explore**	**Summarize**
Approx Time: 2 minutes Cycle 1: I will reread each vocabulary word for the class and will have volunteers tell us what they remember about the words. I will give each group a set of Cuisenaire Rods. I will tell them I want them to explore the relationships among the different lengths and try to find equivalent fractions. I will tell them to record their thoughts and any relationships they see in their logs.	**Approx. Time: 5-10 minutes** Cycle 1: I will monitor students as they use the Cuisenaire Rods to explore relationships. I will ask the following questions: 1. Can you show me a rod that is 1/3 the size of another? 2. Can you show that same relationship using different blocks? 3. What do we know about equivalent fractions? *As the students continue to explore the Cuisenaire Rods in their groups, I	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will begin the "summarize" segment with a think-pair-share. The question will be "What did you notice about the lengths of the Cuisenaire Rods?" On the Promethean Board, students will draw and explain the relationships they noticed between the pattern blocks. I will make sure the student presentations lead the class to the big mathematical ideas. If students do not make this connection, I will point out

	will be selecting students to share their findings with the class. Students will be selected if they have discovered something unique, or if their findings help bring out the big mathematical ideas.	that "equivalent" shapes can be made by combining shorter lengths. For example, 1/3 can also be depicted using 2/6. I will then give the students 3-5 minutes to draw representations in their logs.
Launch **Approx Time: 2 minutes** Cycle 2: I will tell the students that they are going to continue to explore the relationships between pattern blocks, but they will now need to use the words "fraction," "equivalent," "numerator," and "denominator" as they explore and discuss the relationships. I will give them each a paper with questions that help guide their explorations.	**Explore** **Approx. Time: 10-15 minutes** Cycle 2: I will monitor students as they continue to use the pattern blocks to explore relationships. I will guide them to use the words "fraction," "equivalent," "numerator," and "denominator" in their conversations and explanations. I will ask the following questions as I monitor each group: 1. How many green triangles are in one blue rhombus? The green triangle is what fraction of the blue rhombus? What part of the fraction is the numerator? What does the numerator in this fraction mean or represent? What part of the fraction is the denominator? What does the denominator in this fraction mean? 2. How many green triangles are in one red trapezoid? The green triangle	**Summarize** **Approx. Time: 10-15 minutes** Cycle 2: This time we will connect the fraction names to the shapes. For example, if the yellow hexagon is our whole, the red trapezoid is 1/2, the blue rhombus is 1/3, and the green triangle is 1/6. However, if the red trapezoid is one whole, then the green triangle is 1/3. I will ask the students questions like "If the red trapezoid is one whole, how much is the blue rhombus? The yellow hexagon? I will then give the students time to write a summary and draw representations of what they learned about fraction relationships in their logs.

	is what fraction of the red trapezoid? What part of the fraction is the numerator? What does the numerator in this fraction mean or represent? What part of this fraction is the denominator? What does the denominator in this fraction mean? *I will once again be looking for students who have strong ideas to share with the class during the summarize portion of the lesson.	
Accommodations for Diverse Learners: English as a Second Language: I will write the words "fraction," "equivalent," "numerator," and "denominator" on the word wall so the EL students can become familiar with the form of the words.	English as a Second Language: I will take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	English as a Second Language: I will make sure that through the discussion the correct definitions of the focus words are stated clearly.
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Are there enough pattern blocks for each group? As a way to assess each child individually, I will give an additional	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *Fractions can be described as parts

task that they must solve by themselves. I will remind the students to use what they have learned through their exploration.		of a whole. *You can group smaller pattern blocks together to make them "equivalent" to larger blocks. *The denominator of a fraction shows us how many pieces the whole is split into. *The numerator of a fraction tells us how many of those pieces we are talking about.

Lesson 2 Activity

Fraction Fun

Jackson had a pizza that was cut into 8 slices. He ate 4 of them. What fraction of the pizza did Jackson eat?

What is the simplest way to write this fraction?

What if Josh ate the same amount of pizza as Jackson, but his pizza was only cut into 6 slices. What fraction of the pizza did Josh eat?

Name two fractions that are equivalent to each of the following fractions. Draw pictures of the Cuisenaire Rods to show that each set are equivalent.

1/3 _______________ _______________

4/6 _______________ _______________

3/4 _______________ _______________

2/8 _______________ _______________

Lesson 2 Homework

Fractions Homework

Name two fractions that are equivalent to each of the following fractions. Draw pictures to show that each set are equivalent.

3/9 ________________ ________________

1/4 ________________ ________________

3/6 ________________ ________________

6/10 ________________ ________________

Fractions Lesson 3

Topic: Comparing and Ordering Fractions	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/9/10**

Goals: Content Goal: Students will explore how to compare and order fractions by finding equivalent fractions. They will explore the need for finding common denominators in order to be able to compare fractions. Vocabulary Goal: Students will use the words "fraction," "equivalent," "common denominator," and “least common denominator” in conversation and in their logs.	**Materials Needed:** Promethean Board, student logs, pencils, activity sheet, computers
State Core Connection: Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.	

Description of the Mathematical Task: Students will draw pictures to help them compare and order fractions.

Launch	**Explore**	**Summarize**
Approx Time: 2 minutes Cycle 1: I will reread each of the previous vocabulary word for the class. I will then read the new vocabulary words to the class: “common denominators” and “least common denominator.” I will encourage them to think about these terms as they explore.	**Approx. Time: 5-10 minutes** Cycle 1: I will pass out the tasks to the students and read it aloud to them. I will monitor students as they work out the two problems on Side A. I will ask the students to defend their responses. I will keep asking students to explain how they know which fraction is larger. Students will be selected to explain solutions if they have discovered	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will begin the "summarize" segment with a think-pair-share. The question will be "When you were trying to compare the different sized pieces, what problems did you face?” I will follow that up with another question, "What were some things you had to do in order to be able to compare the different pieces?”

	something unique, or if their findings help bring out the big mathematical ideas.	On the Promethean Board, students will draw and explain the relationships they noticed between the pattern blocks. I will make sure the student presentations lead the class to the big mathematical ideas. If students do not make this connection, I will point out that in order to compare the fractions, they needed to make sure they were comparing the same size pieces. This can be done by finding common denominators.
Launch **Approx Time: 2 minutes** Cycle 2: I will tell the students that they are going to continue to compare different fractions. I will encourage them to use the vocabulary "common denominator" in their discussions.	**Explore** **Approx. Time: 10-15 minutes** Cycle 2: I will monitor students as they continue to work on Side B of the task. I will guide them to use the words "common denominator" and "least common denominator" in their conversations and explanations. *I will once again be looking for students who have strong ideas to share with the class during the summarize portion of the lesson.	**Summarize** **Approx. Time: 10-15 minutes** Cycle 2: This time we will go into more depth on how to find common denominators. One way is to find the least common multiple by listing out the multiples of the denominators. Another way is to multiply two denominators together. We will talk about the differences between common denominators and the least common denominator.
Accommodations for Diverse Learners:		

English as a Second Language: I will write the words "common denominator," and "least common denominator" on the word wall so the EL students can become familiar with the form of the words.	English as a Second Language: I will take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	English as a Second Language: I will make sure that through the discussion the correct definitions of the focus words are stated clearly.
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Do the students have pencils and the task? As a way to assess each child individually, I will give an additional task as homework that they must solve by themselves. I will remind the students to use what they have learned through their exploration.	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *When comparing fractions, you must make sure you are comparing same-size pieces. *In order to find same-size pieces, you must find common denominators. *Common denominators can be found by drawing pictures, or multiplying two denominators together. They can also be found by listing the multiples of the denominators to find the least common denominator. Other ways to find the least common multiple will also work. *Once the denominators are the same, compare numerators to determine

		which fraction is larger.

Side A

Emily and Kevin each had the same amount of cookie dough. Emily made 9 small cookies and ate 5 of them. Kevin made 3 larger cookies and ate 2 of them. What fraction of the cookies did each child eat? Who ate more? How do you know?

Sarah had a pan of brownies that was cut into 8 equal pieces. She gave away 5 of them. Joyce had a pan of brownies that was the same size as Sarah's, but Joyce's pan of brownies was cut into 12 pieces. She gave away 7 of them. What fraction of the brownies did each girl give away? Who gave away more? How do you know?

Side B

Jake had a pizza that was cut into 10 equal slices. He ate 4 of them. Sam had a pizza that was the same size as Jake's, but Sam's pizza was cut into 8 equal slices. He ate 3 of them. What fraction of the pizza did each boy eat? Who ate more? How do you know?

Which is larger:

6/7 or 8/9

How do you know?

Order these fractions from least to greatest:

2/3 4/9 5/6 5/9

Lesson 3 Homework

Comparing/Ordering Fractions Homework

1. Which is larger:

6/9 or 8/12

How do you know?

2. Which is larger:

3/8 or 5/9

How do you know?

3. Which is larger:

6/12 or 5/7

How do you know?

4. Order these fractions from least to greatest:

2/9 4/12 1/6 1/3

5. Order these fractions from least to greatest:

3/8 3/4 1/2 5/8

Fractions Lesson 4

Topic: Improper Fractions-Mixed Numbers	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/15/10**
Goals: Content Goal: Students will explore different ways to represent the same fractional amount. They will learn to convert improper fractions to mixed numbers and mixed numbers into improper fractions. Vocabulary Goal: Students will use the words "improper fraction" and "mixed number" in conversation and in their logs. **State Core Connection:** Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.		**Materials Needed:** Promethean Board, student logs, pencils, pattern blocks, activity sheet, computers	

Description of the Mathematical Task: Students will explore improper fractions and mixed numbers. They will try to create a method for converting between the two.

<u>**Launch**</u>	<u>**Explore**</u>	<u>**Summarize**</u>
Approx Time: 2 minutes Cycle 1: I will reread each of the previous vocabulary word for the class. I will then read the new vocabulary words to the class: "improper fraction" and "mixed number." I will encourage them to think about these terms as they explore.	**Approx. Time: 5-10 minutes** Cycle 1: I will pass out the tasks to the students and read it aloud to them. I will monitor students as they work out the problem on the first side of the paper. I will ask the students to draw pictures to explain and defend their responses. They may also manipulate pattern blocks to help them. Students will be selected to explain solutions if they have	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will begin the "summarize" segment with a think-pair-share. The question will be "When you were trying to compare the different sized pieces, what problems did you face?" I will follow that up with another question, "What were some things you had to do in order to be able to compare the different pieces?"

	discovered something unique, or if their findings help bring out the big mathematical ideas.	On the Promethean Board, students will draw and explain the relationships they noticed between the pattern blocks. I will make sure the student presentations lead the class to the big mathematical ideas. If students do not make this connection, I will point out that in order to compare the fractions, they needed to make sure they were comparing the same size pieces. This can be done by finding common denominators.
Launch **Approx Time: 2 minutes** Cycle 2: I will tell the students that they are going to continue to work with mixed numbers and improper fractions. I will explain that the back side of the paper has them create a method for converting fractions back and forth between the two kinds.	**Explore** **Approx. Time: 10-15 minutes** Cycle 2: I will monitor students as they continue to work on Side B of the task. I will guide them to use the words "improper fraction" and "mixed number" in their conversations and explanations. *I will once again be looking for students who have strong ideas to share with the class during the summarize portion of the lesson.	**Summarize** **Approx. Time: 10-15 minutes** Cycle 2: This time we will go into more depth connecting the procedural methods for converting between mixed numbers and improper fractions and the more conceptual, concrete way of thinking about it. This will be done through pictures.
Accommodations for Diverse Learners:	English as a Second Language: I will	English as a Second Language: I will

English as a Second Language: I will write the words "mixed number," and "improper fraction" on the word wall so the EL students can become familiar with the form of the words.	take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	make sure that through the discussion the correct definitions of the focus words are stated clearly.
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Do the students have pencils and the task? As a way to assess each child individually, I will give an additional task as homework that they must solve by themselves. I will remind the students to use what they have learned through their exploration.	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *Improper fractions and mixed numbers can represent equivalent amounts. *You can convert between the two kinds of fractions. *There is a procedural method for converting them. This method is logical and makes a lot of sense, especially when the process is seen through pictures.

Lesson 4 Activity

Name: ______________________________ Date: ________________________________

Some sixth graders were having a pizza party. The pizzas were cut into 6 slices each. The kids ate a total of 22 slices of pizza. How much pizza was eaten? Is there more than one way to write the amount of pizza that was eaten?

Use the space below to draw pictures to explain your answer:

Change the following improper fraction into a mixed number:

$$\frac{16}{3}$$

Create a method for changing improper fractions into mixed numbers:

Change the following mixed number into an improper fraction:

$$3\ \frac{2}{5}$$

Create a method for changing improper fractions into mixed numbers:

Name: ______________________________ Date: ____________________________

Homework

Change the following improper fractions into mixed numbers:

$\frac{14}{5}$

$\frac{18}{3}$

$\frac{44}{6}$

$\frac{9}{2}$

Change the following mixed numbers into improper fractions:

$2\ \frac{5}{8}$

$7\ \frac{1}{4}$

$4\ \frac{2}{5}$

$6\ \frac{3}{10}$

Fractions Lesson 5

Topic: Adding and Subtracting Fractions with Like Denominators	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/17/10**

Goals: Content Goal: Students will be able to add fractions with like denominators. Vocabulary Goal: Students will use all of the previous vocabulary words listed on the word wall in conversation and in their logs.	**Materials Needed:** Promethean Board, student logs, pencils, activity sheet, computers, homework paper
State Core Connection: Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.	

Description of the Mathematical Task: Students will explore the idea of adding and subtracting fractions. They will solve word problems involving addition and subtraction of fractions with like denominators.

Launch	**Explore**	**Summarize**
Approx Time: 2 minutes Cycle 1: I will reread each of the previous vocabulary word for the class. I will encourage them to think about these terms as they explore.	**Approx. Time: 5-10 minutes** Cycle 1: I will pass out the tasks to the students and read it aloud to them. I will monitor students as they work out the problems on the first side of the paper. I will ask the students to draw pictures to explain and defend their responses. Students will be selected to explain solutions if they have discovered something unique, or if their findings help bring out the big mathematical ideas.	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will begin the "summarize" segment by having students come up to explain each of the four questions on Side A. On the Promethean Board, students will draw and explain how they solved the problems. I will make sure the student presentations lead the class to the big mathematical ideas. If students do not make this connection, I will point out that when adding and

		subtracting fractions with like denominators, the denominator must stay the same because you are talking about the same size pieces. You simply need to add the numerator and then simplify the results.
Launch **Approx Time: 2 minutes** Cycle 2: I will tell the students that they are going to continue to work with adding and subtracting fractions with like denominators. I will read the word problem on Side B to the class.	**Explore** **Approx. Time: 10-15 minutes** Cycle 2: I will monitor students as they continue to work on Side B of the task. I will guide them to use the words from the word wall in their conversations and explanations. *I will once again be looking for students who have strong ideas to share with the class during the summarize portion of the lesson.	**Summarize** **Approx. Time: 10-15 minutes** Cycle 2: This time we will get a lot more practice adding and subtracting fractions with like denominators. I will select a few students who have solved the problem in different ways to present their findings on the board. After we have gone over the solution, I will write the following question on the board: “Why to the denominators stay the same when adding and subtracting fractions?” I will give them time to respond to this question in their fraction logs. I will also let them write about any other ideas relating to fractions that they would like to write about.
Accommodations for Diverse Learners: English as a Second Language: I will	English as a Second Language: I will take special care to use these words in	English as a Second Language: I will make sure that through the discussion

point out all the vocabulary words on the word wall so the EL students can become familiar with the form of the words.	conversation with the EL students and to make sure they understand the meanings of the words.	the correct definitions of the focus words are stated clearly.
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Do the students have pencils and the task? As a way to assess each child individually, I will give an additional task as homework that they must solve by themselves. I will remind the students to use what they have learned through their exploration.	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *When adding and subtracting fractions with like denominators, the denominators must stay the same. *The denominators stay the same because you cannot change the size of the piece you are referring to. *You simply add or subtract the numerators and keep the denominator the same.

Activity 5

Side A

Julie made a pan of brownies and cut them into 15 pieces. She ate 6 of them that day. The next day, she ate 4 more. What fraction of the whole pan did Julie eat?

Two boys were eating pizza. Ryan ate 5/6 of a pizza, and Thomas ate 4/6 of a pizza. How much pizza did the boys eat all together?

My dog is 10/12 years old. My cat is 7/12 years younger than my dog. How old is my cat?

Amber lives 5/8 of a mile from the school. She has already walked 3/8 of a mile toward her home. How much farther must she walk to get home?

Side B

Kevin was going to go to the swimming pool, but he needed to run some errands around town first. He started at his house and drove 5/8 of the way to the pool for his first stop. Then he drove 2/8 of the way back towards his house for his second stop. For his third stop, he drove 4/8 of the way, once again going toward the swimming pool. How far is Kevin away from the swimming pool when he's at his third stop?

Lesson 5 Homework

Homework

Add or subtract. Put your answers in simplest form.

1) 2/7 + 3/7 =

2) 3/5 + 1/5 =

3) 2/9 + 4/9 =

4) 3/10 + 2/10 =

5) 4/8 + 2/8 =

6) 9/12 – 6/12 =

7) 7/8 – 3/8 =

8) 5/7 – 1/7 =

9) 8/9 – 5/9 =

10) 6/6 – 2/6 =

Fractions Lesson 6

Topic: Adding and Subtracting Fractions with Different Denominators	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/22/10**

Goals: Content Goal: Students will be able to add fractions and subtract with different denominators. Vocabulary Goal: Students will use all of the previous vocabulary words listed on the word wall in conversation and in their logs. They will also be able to use the words "common denominator" and "least common denominator" in correct contexts.	**Materials Needed:** Promethean Board, student logs, pencils, activity sheet, computers, homework paper
State Core Connection: Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.	

Description of the Mathematical Task: Students will explore the idea of adding and subtracting fractions that have different denominators. They will solve word problems involving addition and subtraction of fractions with unlike denominators.

Launch	**Explore**	**Summarize**
Approx Time: 2 minutes Cycle 1: I will have the students read each of the previous vocabulary words on the word wall silently to themselves. I will tell them that two of the words will be particularly helpful today, and I want them to think about which two terms I	**Approx. Time: 5-10 minutes** Cycle 1: I will pass out the tasks to the students. I will monitor students as they work out the problems on the first side of the paper. I will ask the students to draw pictures to explain and defend their responses. Students	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will begin the "summarize" segment by having a think-pair-share. As a new twist, I am going to have the students write in their logs before they share their thoughts with a partner. The

might be referring to through out their explorations.	will be selected to explain solutions if they have discovered something unique, or if their findings help bring out the big mathematical ideas.	questions will by "Why are the words 'common denominator' and 'least common denominator' so important in today's activity?" Following the activity, students will draw and explain how they solved the problems on the Promethean Board. I will make sure the student presentations lead the class to the big mathematical ideas. If students do not make this connection, I will point out that when adding and subtracting fractions with like denominators, the denominator must stay the same because you are talking about the same size pieces. You simply need to add the numerator and then simplify the results.
Accommodations for Diverse Learners: English as a Second Language: I will point out all the vocabulary words on the word wall so the EL students can become familiar with the form of the words.	English as a Second Language: I will take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	English as a Second Language: I will make sure that through the discussion the correct definitions of the focus words are stated clearly.
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do?	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will

E: Do the students have pencils and the task? As a way to assess each child individually, I will give an additional task as homework that they must solve by themselves. I will remind the students to use what they have learned through their exploration.	assessment. (See attached task)	also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *When adding and subtracting fractions with unlike denominators, common denominators must be found. *The denominators must be the same so the size piece is the same. You cannot add thirds to fourths. *Once you have found common denominators, you simply add or subtract the numerators and keep the denominator the same. *Some problems will also require you to simplify your answer.

Lesson 6 Activity

Name: ______________________________ Date: ______________________________

Sam, Sally, Jane, and Josh are all training for a triathlon. Solve the following problems:

Sam rode his bike 1/8 of a mile and ran another 3/4 of a mile. How far did he travel?

Sally walked 3/5 of a mile before lunch and 1/3 of a mile after lunch. How far did she walk in all?

Jane swam 5/6 of a mile on Friday and 3/4 of a mile on Saturday. How far did she swim on these two days?

Josh jogged 1/4 of a mile and then rode his bike 4/5 of a mile. How far did he travel?

Tom bought a board that was 7/8 of a yard long. He cut off 1/2 of a yard. How much was left?

Emma bought 8/9 of a pound of chocolates and ate 1/3 of a pound. How much was left?

Don bought 3/4 of a pound of jellybeans and 6/9 of a pound of gummy bears. How many more pounds of jellybeans did Dan buy than gummy bears?

Sarah has an apple and a banana. The apple weighs 5/6 of a pound. The banana weighs 3/4 of a pound. Which one is heavier, and how much more does it weigh?

Fractions Lesson 7

Topic: Adding Mixed Numbers	Time Frame: 60 minutes	Grade Level: 6	Date: 3/24/10
Goals: Content Goal: Students will be able to add mixed numbers. Vocabulary Goal: Students will use all of the previous vocabulary words listed on the word wall in conversation and in their logs. **State Core Connection:** Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.		**Materials Needed:** Promethean Board, student logs, pencils, activity sheet, computers, homework paper	

Description of the Mathematical Task: Students will explore adding mixed numbers. They will solve word problems involving addition of mixed numbers.

Launch	**Explore**	**Summarize**
Approx Time: 2 minutes Cycle 1: I will ask a student to summarize the big mathematical ideas from our last class. These involved finding common denominators when adding and subtracting fractions. I will then tell them we will be adding mixed numbers today. I will encourage them to think about the vocabulary terms as they explore. I will also encourage them to draw pictures to explain.	**Approx. Time: 5-10 minutes** Cycle 1: I will pass out the tasks to the students and read it aloud to them. I will monitor students as they work out the problems on the first side of the paper. I will ask the students to draw pictures to explain and defend their responses. Students will be selected to explain solutions if they have discovered something unique, or if their findings help bring out the big mathematical ideas.	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will begin the "summarize" segment by having students come up to explain each of the four questions on Side A. On the Promethean Board, students will draw and explain how they solved the problems. I will make sure the student presentations lead the class to the big mathematical ideas.

Accommodations for Diverse Learners: English as a Second Language: I will point out all the vocabulary words on the word wall so the EL students can become familiar with the form of the words.	English as a Second Language: I will take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	English as a Second Language: I will make sure that through the discussion the correct definitions of the focus words are stated clearly.
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Do the students have pencils and the task? As a way to assess each child individually, I will give an additional task as homework that they must solve by themselves. I will remind the students to use what they have learned through their exploration.	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *When adding mixed numbers, you can add the whole numbers and then add the fractions. *Sometimes when adding the fractions, your result is larger than one whole. If this is the case, you must regroup. *Another way to add mixed numbers is to change them into improper fractions. Add, and then change them back to mixed numbers.

Lesson 7 Activity

Name: ______________________________ Date: _______________________________

Adding Fractions

Draw pictures to show how you solve each problem:

1. Jack and Jill brought their leftover pies to an after-Thanksgiving party. Jack brought 4 1/5 pies, and Jill brought 3 2/5 pies. How many pies did they have all together?

2. Aaron and Allison were eating cupcakes. Aaron ate 2 1/4 cupcakes, and Allison ate 1 3/8 cupcakes. How many cupcakes did they eat all together?

3. Ben and Brenda had a pizza party. Ben ate 5 ¾ of a pizza, and Brenda ate 3 ½ pizzas. How many pizzas did they eat total?

4. Kayla and Kyle bought some fabric. Kayla bought fabric that was 5 7/8 yards long, and Kyle bought fabric that was 3 5/6 yards long. How many yards of fabric did the two of them buy all together?

How many different ways can you think of to add fractions?

Fractions Lesson 8

Topic: Subtracting Mixed Numbers	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/26/10**
Goals: Content Goal: Students will explore subtraction of mixed numbers. Students will only subtract mixed numbers that do not require regrouping. Vocabulary Goal: Students will be conscious of the mathematics vocabulary that they are using and that their group members use. **State Core Connection:** Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.		**Materials Needed:** Promethean Board, student logs, pencils, activity sheet, computers	

Description of the Mathematical Task: Students will explore subtraction of mixed numbers. They will only be given questions that do not require regrouping today. They also will be asked to create their own subtraction word problem.

Launch	**Explore**	**Summarize**
Approx Time: 2 minutes Cycle 1: I will read the task aloud to the students. I will then ask them to draw a box in their fraction logs. They are to write down the names of their partners and make a tally mark each time they hear a partner use a word wall vocabulary word correctly. We will then read aloud each word wall vocabulary word together as a class.	**Approx. Time: 5-10 minutes** Cycle 1: I will pass out the tasks to the students and read it aloud to them. I will monitor students as they work out the problems on the first side of the paper. I will ask the students to draw pictures to explain and defend their responses. Students will also write their own subtraction word problems. They will then find other students to solve their problems. Students will be selected to explain solutions if they have	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will select students to draw and explain the way they solved the questions. I will make sure the student presentations lead the class to the big mathematical ideas.

	discovered something unique, or if their findings help bring out the big mathematical ideas.	
Accommodations for Diverse Learners: English as a Second Language: I will state each word on the word wall so the EL students can become familiar with the form of the words.	English as a Second Language: I will take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	English as a Second Language: I will make sure that through the discussion the correct definitions of the focus words are stated clearly.
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Do the students have pencils and the task? As a way to assess each child individually, I will give an additional task as homework that they must solve by themselves. I will remind the students to use what they have learned through their exploration.	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *When subtracting mixed numbers in which the fraction part of the first number is larger than the fraction part of the second number, you simply need to subtract the whole numbers and then subtract the fractions. *This may require you to find common denominators. *This can also be done by changing

		the mixed numbers into improper fractions.

Lesson 8 Activity

Name: ______________________________ Date: _____________________________

1. Crystal bought 4 ¾ pounds of candy. She gave 2 1/6 pounds away. How much candy does she have left?

2. Greg walked 5 7/8 miles, and Sammy walked 1 2/3 miles. How much farther did Greg walk than Sammy?

3. Jonathan had rope that was 13 2/3 feet long. He cut off 4 1/5 feet. How long is the remainder of his rope?

Write your own word problem involving subtraction of mixed numbers. Then find another person in the class to solve it.

Word Problem Creator: ________________ Word Problem Solver: _________________

Fractions Lesson 9

Topic: Subtracting Mixed Numbers with Regrouping	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/29/10**

Goals: Content Goal: Students will explore subtraction of mixed numbers where regrouping is required. Vocabulary Goal: Students will be conscious of the mathematics vocabulary that they are using and that their group members use.	**Materials Needed:** Promethean Board, student logs, pencils, activity sheet, computers
State Core Connection: Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.	

Description of the Mathematical Task: Students will explore subtraction of mixed numbers. They will only be given questions that require regrouping today. They also will be asked to create their own subtraction word problem.

Launch	**Explore**	**Summarize**
Approx Time: 10 minutes Cycle 1: I will begin by writing the following word problem on the board: "Sharon made ______ pies. Her friend ate _______ pies. How many pies are left?" I will ask the students if they know what operation they must use. I will then replace the blanks with various numbers that get progressively more	**Approx. Time: 20-30 minutes** Cycle 1: I will pass out the tasks to the students and read it aloud to them. I will monitor students as they work out the problems on the first side of the paper. I will ask the students to draw pictures to explain and defend their responses. Students will also write their own subtraction word problems hat require regrouping. They will then find other students to solve their problems.	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will select students to draw and explain the way they solved the questions. I will make sure the student presentations lead the class to the big mathematical ideas.

difficult. They will be: 7-3=4 7 5/6 – 3 4/6 = 4 1/6 7 11/12 – 3 4/6 = 4 3/12 = 4 ¼ 7 1/5 – 3 3/5 I will then ask the students to write in their fraction logs why these last numbers are more difficult than the other numbers I used. We will then do a think-pair-share to have the students share why it's more difficult.	Students will be selected to explain solutions if they have discovered something unique, or if their findings help bring out the big mathematical ideas.	
Accommodations for Diverse Learners: English as a Second Language: I will state each word on the word wall so the EL students can become familiar with the form of the words.	English as a Second Language: I will take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	English as a Second Language: I will make sure that through the discussion the correct definitions of the focus words are stated clearly.
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Do the students have pencils and the task? As a way to assess each child individually, I will give an additional task as homework that they must solve	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include: *When subtracting mixed numbers in which the fraction part of the first

by themselves. I will remind the students to use what they have learned through their exploration.		number is smaller than the fraction part of the second number, you must regroup in order to be able to subtract. *This may require you to find common denominators. *Another helpful strategy is to change the mixed numbers into improper fractions, subtract, and then change back to mixed numbers.

Lesson 9 Activity

Name: ______________________________ Date: _______________________________

1. Kaden bought 4 ¼ pounds of candy. He gave 2 ¾ pounds away. How much candy does he have left?

2. Terry walked 5 1/6 miles, and Karen walked 1 2/3 miles. How much farther did Terry walk than Karen?

3. Jacob had rope that was 9 2/3 feet long. He cut off 4 4/5 feet. How long is the remainder of his rope?

Write your own word problem involving subtraction of mixed numbers. Make the fraction part of the first mixed number smaller than the fraction part of the second mixed number. Solve the question yourself, then find another person in the class to solve it.

Word Problem Creator: ________________ Word Problem Solver: _________________

Fractions Lesson 10

Topic: Subtracting Mixed Numbers with Regrouping	**Time Frame: 60 minutes**	**Grade Level: 6**	**Date: 3/31/10**
Goals: Content Goal: Students will explore a task that requires use of all the skills they have learned in this fraction unit, involving adding and subtracting fractions and mixed numbers, changing between mixed numbers and improper fractions, etc. Vocabulary Goal: Students will be conscious of the mathematics vocabulary that they are using and that their group members use.		**Materials Needed:** Promethean Board, student logs, pencils, activity sheet, computers	
State Core Connection: Utah State Mathematics Core Standard 1.2: Explain relationships and equivalencies among numbers.			

Description of the Mathematical Task: Students will explore subtraction of mixed numbers. They will only be given questions that require regrouping today. They also will be asked to create their own subtraction word problem.

Launch	**Explore**	**Summarize**
Approx Time: 2 minutes Cycle 1:	**Approx. Time: 20-30 minutes** Cycle 1: I will pass out the tasks to the students and read it aloud to them. I will monitor students as they work out the problem. I will ask the students to draw pictures to explain and defend their responses. Students will be selected to explain solutions if they have discovered something unique, or if their findings help bring out the big mathematical	**Approx. Time: 10-15 minutes** Cycle 1: After the students have explored, I will select students to draw and explain the way they solved the questions. I will make sure the student presentations lead the class to the big mathematical ideas.

	ideas.	
Launch **Approx Time: 2 minutes** Cycle 2: I will tell the students that they will be writing in their fraction logs. I will ask them to think about all of the different steps and manipulations they were required to use/do in order to solve the problem. I will then give them time to write in their logs.	**Explore** **Approx. Time: 10-15 minutes** Cycle 2: I will monitor students as they write in their logs. I will ask guiding questions if the students need help. *I will once again be looking for students who have strong ideas to share with the class during the summarize portion of the lesson.	**Summarize** **Approx. Time: 10-15 minutes** Cycle 2: I will ask students to share all of the things they needed to do in order to solve the question. I want to point out that they were required to use all of the skills we have learned this unit, including: *Finding common denominators *Finding equivalent fractions *Changing improper fractions to mixed numbers and vice-versa *Adding fractions *Subtracting fractions *Adding and subtracting mixed numbers *Regrouping with addition *Regrouping with subtraction
Accommodations for Diverse Learners: English as a Second Language: I will state each word on the word wall so the EL students can become familiar	English as a Second Language: I will take special care to use these words in conversation with the EL students and to make sure they understand the meanings of the words.	English as a Second Language: I will make sure that through the discussion the correct definitions of the focus words are stated clearly.

with the form of the words.		
Assessment (Launch) T: Have I stated the instructions clearly? L: Do the students know what to do? E: Do the students have pencils and the task? As a way to assess each child individually, I will give an additional task as homework that they must solve by themselves. I will remind the students to use what they have learned through their exploration.	**Assessment** (Explore) In addition to the student notes and visual representations in logs, this individual task will serve as a form of assessment. (See attached task)	**Assessment** (Summarize) After I collect the individual assessments, I will allow students to share their ideas with the class. I will also clearly explain the big mathematical ideas that were addressed in this lesson. These include all of the skills listed above in the Cycle 2 Summarize section. I will then ask the students to complete a Venn diagram with the words "mixed numbers" and "improper fractions" over each circle.

Lesson 10

Name: ________________________________ Date: ____________________________

Janet was building a door from a piece of wood, but she kept getting the measurements wrong. First, she thought the piece was too long, so she cut 3/4 of a foot off the board. Then she realized she cut it too short, so she nailed on another piece of wood that was 1/3 foot long. Then she decided she wanted to door to be a bit shorter, so she cut off another 1/2 foot. The final door was 6 2/3 feet. How long was the piece of wood that she started with?

Appendix E

Codes, Categories, and Themes

Use of Mathematical Processes

Communication

1 Student thinks aloud
2 Student shares an incomplete thought
3 Student asks a question
4 Student compares/contrasts ideas
5 Student repeats an explanation in a new way

Justification

6 Student gives correct answer, with clear justification
7 Student gives correct answer, with unclear justification
8 Student gives incorrect answer, with attempted justification
9 Student appeals to authority (someone told me to . . .)

Representations

10 Student draws a picture or other representation

Connections

11 Student verbally connects thinking to their pictures or written words
12 Student expresses a fraction as a decimal (or vice versa)
13 Student expresses connection to previous knowledge (e.g., I learned this in 5th grade)

Content Knowledge

Conceptual Understanding

14 Student demonstrates conceptual understanding
15 Student expresses a new understanding of a concept (oooooooooh)
16 Student recognizes the appropriate operation to use when solving a problem
17 Student demonstrates lack of prerequisite knowledge
18 Student adds numerators AND denominators

Procedural Understanding

19 Student demonstrates procedural understanding

Mathematical Accuracy

20 Correct Answer with extensive prompting
21 Correct Answer with minimal prompting
22 Correct Answer with no prompting
23 Incorrect Answer with prompting
24 Incorrect Answer without prompting

Mathematical Confidence
25 Student expresses confidence in mathematical understanding
26 Student appears unsure in response (answers in question form)
27 Student responds with hesitance (ummmm)
28 Student claims to not know/understand

Mathematics Vocabulary
29 Student uses mathematics vocabulary correctly
30 Student expresses surface understanding of a vocabulary word (denominator-number on the bottom)
31 Student uses imprecise words in place of mathematics vocabulary ("top ones"-numerator)
32 Student expresses confusion about a mathematics vocabulary word
33 Reads or says fraction names incorrectly (nine-twoths)
34 Student incorrectly reads a decimal

Student Use of Metacognition
35 Student recognizes a gap in his/her understanding
36 Student recognizes an error and corrects him/herself

Other
37 Student was inattentive

Appendix F

Daily Task Rubric

4	The student has answered the question correctly and demonstrated advanced thinking through his or her work.
3	The student has answered the question correctly.
2	The student has attempted to answer the problem and shows progress toward the correct answer.
1	The student has attempted to answer the problem but shows insufficient progress toward the correct answer.
0	No attempt has been made to answer the question.

Additional Codes

a	Student has received extensive prompting
b	Student has received minimal prompting
c	Student shows answer, but not enough work to prove he or she understand (maybe copied answer from neighbor)